Paraxial Ray Tracing

Javier Alda
University Complutense of Madrid, Madrid, Spain

Josep Arasa
University Polytechnique of Catalonia, Terrassa, Barcelona, Spain

INTRODUCTION

Ray tracing translates into a graphic language the algebra supporting geometrical optics. Besides, it makes possible to know the spatial location of light trajectories. Moreover, the graphical nature of ray tracing allows dropping sign conventions and changeable references. Then, ray tracing becomes the closest and most intuitive approach to the real thing underlying the propagation of light through optical systems.

Paraxial optics is described and analyzed elsewhere in textbooks and reference books. The paraxial regime allows a very well-established method of analysis of optical systems. It produces relations among the location and size of the image and the object transformed by an optical system. It defines optical parameters, as the focal length or the F#, characterizing optical elements and their combinations. These parameters are used, disregarding their paraxial origin, to determine the behavior of actual, or real, optical systems. All these analytical results can also be obtained by a graphical method based on a few and well-defined rules. This graphical treatment is the paraxial ray tracing.

In this contribution, we begin by presenting the very basic rules of paraxial optics. Then, we explain a step-by-step method to solve problems by using paraxial ray tracing. ''Easy'' rays and auxiliary rays are presented and traced. Some examples will be explained with detail for those readers approaching this topic for the very first time. The paraxial ray tracing will find one of its practical uses in the calculation of apertures and fields.

CONSTRAINTS, LIMITATIONS, AND ADVANTAGES OF PARAXIAL RAY TRACING

Paraxial optics[1–7] is described within the geometrical approach (i.e., neglecting the wave nature of light) and when the values of the angles involved in the description of the propagation of light are small enough to apply the paraxial approximations; that is, the sine and tangent of the angles are substituted by the value of the angle itself (in radians), and the cosine of the angle is assumed to be 1. Within the paraxial approach, the angles are usually assumed to be below 20°, although this limit is not a standard. This restriction applies to incidence, reflection, and refraction angles and also to the elevation angles with respect to the optical axis. However, as soon as we begin to apply the graphical rules of paraxial ray tracing, we will realize that this limitation is apparently dismissed and forgotten. We will trace rays involving angles very well beyond the paraxial regime. The explanation to this apparent inconsistency is supported in paraxial optics itself. With the use of a simple algebra, it is possible to find a paraxial relation transforming the angles of the rays with respect to the optical axis for a given dioptric interface. This relation is known as the Lange equation:

$$n'\sigma' - n\sigma = h\frac{n' - n}{r} \tag{1}$$

where n and n' are the index of refraction at each side of a diopter having a radius r, σ and σ' are the angles of the input and output rays with respect to the optical axis, and h is the height of the impact of the input ray at the input plane (Fig. 1). When substituting the angles by their paraxial relations in terms of the distance from the diopter to the object and image point, s and s', as $\sigma = h/s$ and $\sigma' = h/s'$, we find the following equation:

$$n'\frac{h}{s'} - n\frac{h}{s} = h\frac{n' - n}{r} \tag{2}$$

In this equation, the paraxial limitation is transferred to the value of h. The height of the incidence needs to be small enough to maintain the angle within the paraxial approach. Besides, the paraxial ray tracing uses reference planes attached to the vertex of the optical surfaces. This is equivalent to neglect the sagitta of a dioptric or reflecting surface. On the other hand, this substitution means the use of the tangent of σ and σ', transforming the Lange equation into

$$n'\tan\sigma' - n\tan\sigma = h\frac{n' - n}{r} \tag{3}$$

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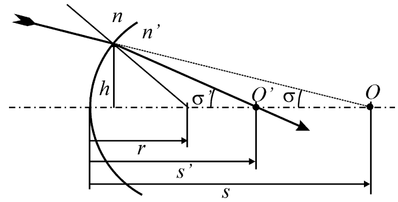

Fig. 1 Definition of the angles and distances when a ray impinges on a curved interface separating two dielectric media.

Then, this equation is still valid when the height of incidence, h, increases beyond the paraxial limit.

Intrinsically related with the previous reasoning and Fig. 1, we have made several nonexplicit assumptions that limit the scope of application of paraxial ray tracing. For example, we have established that light travels from left to right. Then, a backward propagation means to have light traveling from right to left. We have been using the concept of optical axis defined as the straight line containing the centers of curvature of the diopters. Therefore paraxial ray tracing involves the use of tangential, or meridian, planes, i.e., those planes containing the optical axis of the optical system. Another limitation of paraxial ray tracing is related with the graphical layout itself. We will usually draw our ray-tracing problems on a sheet of paper or on a computer graphic template resembling a given meridian plane. Therefore the ray-tracing analysis is a two-dimensional approach to a three-dimensional case. When the optical system is a rotationally symmetric system with the optical axis as its axis of symmetry, the analysis in the meridian plane is a good start to understand the basic behavior of the optical system. In those systems having two orthogonal planes of symmetry intersecting at the optical axis, the paraxial ray tracing can be independently applied to those planes and the results need to be properly combined in a three-dimensional layout.

The media usually considered in paraxial optics are homogeneous, linear, and isotropic. They are characterized by the value of the index of refraction. This means that the trajectories of light, the rays, are straight lines that change their directions when crossing an interface between two media (including reflection on a reflective surface as a special case of change of index of refraction). Paraxial and real ray tracing can be extended to anisotropic media by decoupling the propagation of the ordinary and extraordinary polarization components.

Besides the simplicity of their algebraic relations, paraxial optics describes optical systems behaving within a perfect regime: perfect optical systems transform points into points, planes into planes, and produce an image that is proportional in size to the object. Then, real systems are better as they approach more to the paraxial regime, and real ray tracing always has as a reference the results obtained from the paraxial ray tracing. Indeed, paraxial ray tracing can still be obtained from real ray-tracing algorithms by restricting the analysis within the conditions of paraxial optics, typically by reducing the input pupil and window.

The main advantage of paraxial ray tracing is the intuitive representation and understanding that it produces. The set of rules is limited, simple, and precisely defined. The graphical representation sticks to the real thing and explains the basic behavior of light propagation. Some concepts, as the reversibility of optical trajectories, are easily represented and used. Besides its rigorous layout, paraxial ray tracing allows powerful back-of-the-envelope sketches that are very much appreciated in a preliminary analysis of optical systems.

THE CARDINAL ELEMENTS IN PARAXIAL RAY TRACING

The definition of several characteristic planes and points, the cardinal points, in a centered optical system is of great importance in paraxial optics. Now we will remind the most useful properties of these elements for the case of paraxial ray tracing. The cardinal elements are principal points and planes, nodal points, and focal points and planes.

The principal planes are defined as having a lateral magnification equal to 1, $\beta' = 1$. Then, the height of a ray incident on the object principal plane remains the same at the image principal plane, independently of the angle of arrival to the principal plane. For optical systems considered as thin, both the object and image principal planes coincide. Therefore when a ray arrives to the principal plane of a thin system (lenses or mirrors), it only changes direction maintaining the continuity of the light path. For a compound system having separated principal planes, disregarding what happens in between these planes, the ray will depart the image principal plane at the same height of arrival to the object principal plane.

The nodal points are defined as those points on the optical axis showing an angular magnification of 1, $\gamma' = 1$. When a ray arrives to the object nodal point subtending a given angle with respect to the optical axis, it leaves the image nodal point subtending the same angle than the incoming ray. When the indices of refraction at the object and image spaces are the same, then the nodal points coincide with the principal points (the principal points are the intersection of the principal planes with the optical axis). Besides, if the optical system can be considered as thin, then those rays incident at the intersection of the optical system and the optical axis do not change their trajectories.

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The image focal point is the image of an object located at the infinity on the optical axis in the object space. Then, a ray parallel to the optical axis needs to pass through the image focal point, usually labeled as F'. Consequently, the image focal plane is the image of a plane located at the infinity in the object space. A bundle of rays coming from an object point at infinite distance is represented as a parallel bundle. They are parallel to the optical axis if the object point is on the axis. For any other point, they form a given angle with respect to the axis. Following the rules of paraxial optics and the behavior of a perfect optical system, all these parallel rays intersect at the same point on the image focal plane. Therefore every point of the image focal plane is the image of a point at the infinity in the object space. As longer is the distance of the given point of the focal plane with respect to the optical axis, the angle subtended by the bundle of parallel rays coming from infinity is larger.

The object focal point, F, is a point on the optical axis that is the object for an image point located at infinite distance on the optical axis in the image space. Therefore a bundle of rays departing from the object focal point, after crossing the optical system, becomes a bundle of parallel rays also parallel to the axis. Following the same reasoning than in the case of the image focal plane, all the points on the object focal plane produce bundles of parallel rays with an angle with respect to the optical axis that increases as the distance between the optical axis and the considered point on the object focal plane increases.

One of the basic rules of geometrical optics establishes that light trajectories are reversible. This means that we can ray-trace backward, from the image space to the object space. This ray tracing has to be performed carefully, understanding that the definitions of object and image focal and principal points have to be properly analyzed for this backward propagation.

RAY TRACING STEP-BY-STEP

In the following, we will use the previously mentioned properties and paraxial results to define a step-by-step graphical method to solve a given problem.

First Step

Translate into a graphical language the characteristics of a given optical system. The graphical layout needs to be obtained from numeric inputs or specifications. The object or image distances, the focal lengths, the radius of curvature and indices of refraction, the thicknesses of the elements, the separations between consecutive optical elements, etc. are given in terms of their values or in terms of inner relations and conditions. Some of these parameters may be unknown and a result for them is what we are looking for. Therefore first of all, we need to locate the available data with respect to a given optical axis. Sometimes it is necessary to calculate the desired parameters by using paraxial optics formula. The final result of this step is a faithful graphical representation of a meridian plane of the optical system showing as much information as possible. The attention should be focused on the location of the focal points, the principal planes, the nodal points (if different from the principal points), the transversal dimensions of optical elements, and the object and image points and sizes. When the optical system is a combination of several optical systems, it is necessary to section the analysis into as many subproblems as many subsystems we have.

Second Step

Translate into graphical language how the solution to the problem is going to be attained. This step depends on the type of problem under consideration. If the location and size of the image has to be found, we may use that, for a perfect optical system, all the rays departing from the object point or plane arrive to the conjugated image point or plane. On the other hand, in a two-dimensional plot, a point is given as the intersection of two straight lines. Then, if we are able to trace a couple of rays from a given point of the object (e.g., its maximum lateral extension), then the intersection of these rays in the image space will locate that point in the image and, after applying the proportionality between object and image, it produces the image of the whole object. When the location of principal or focal points of a combination of optical elements is wanted, then the definitions of these elements need to be applied rigorously to trace only those rays defining the desired parameter. For example, the image focal point will be obtained by tracing a ray parallel to the optical axis and looking for its intersection with the optical axis in the image space of the system.

Third Step

Choose the appropriate rays and trace them through the system. The real trajectories of the rays are usually represented as solid lines, while virtual trajectories are plotted as dashed lines (real trajectories define the actual light paths, and virtual trajectories are extension of the real ones). Typically, the selected rays are those behaving more easily. To properly address this selection, we itemize the following types of ''easy'' rays (Fig. 2):

A. The optical-axis ray. A ray traveling superimposed on the optical axis always remains on the optical

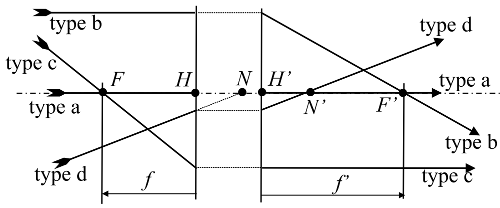

Fig. 2 Ray tracing of the four types of ''easy'' rays.

axis. This trivial ray is very useful when considering points located on the axis because their conjugated points, linked by the object–image relation, are always on the axis, and therefore we would only need another extra ray to obtain their location.

B. The incoming parallel-to-the-axis ray. This ray passes through the image focal point.

C. The outgoing parallel-to-the-axis ray. This ray passes through the object focal point.

D. The nodal ray. This ray arrives to the object nodal point and leaves the system at the image nodal point being parallel to the incoming ray. When the system is immersed in the same index of refraction at both sides and it can be considered as a thin element, this ray incided at the location of the optical element on the axis of the system and it seems to cross the system unperturbed.

Unfortunately, sometimes the ray we are interested in, the problem ray, is not any of the previous ''easy'' rays. Then, it is necessary to use auxiliary rays to help the ray tracing progress (Fig. 3). These auxiliary rays are ''easy'' rays. Typically, the problem ray is plotted with a thicker line than the auxiliary ones.

When the problem ray is in the object space, the auxiliary rays will be ''easy'' rays of the types b, c, or d. The incoming parallel-to-the-axis auxiliary ray (type b) will be traced from the point of intersection of the problem ray with the object focal plane. The auxiliary ray will pass through the image focal point, and the problem ray in the image space has to be parallel to the auxiliary ray in the image space. An auxiliary ray of type c can also be used. The outgoing parallel-to-the-axis auxiliary ray is passing through the object focal point and is selected to be parallel to the problem ray in the object space. Then, in the image space, they have to intersect in the same point of the image focal plane. If the auxiliary ray is a nodal ray, d-type, this is chosen to be parallel to the problem ray, and then in the image space, the auxiliary and the problem rays at the image space intersect at the same point of the image focal plane. The election between these three auxiliary rays will depend on the type of problem and our familiarity with the ray tracing of these ''easy'' rays. The location of the emerging problem ray is obtained after applying the principal planes property that keeps invariant the height of the ray from the object principal plane to the image principal plane.

When the problem ray is in the image space, a similar strategy can be applied to trace auxiliary rays of the types b, c, or d.

Fourth Step

Translate the obtained graphical solution to the expected specifications. This step is inverse to the first one. In this case, the graphical layout may have produced a measurable distance, a defined character of the object or image, or a given trend in the evolution of some parameters with respect to others. The same set of principles, rules and formulas used to build up the graphical situation will be used to analyze the graphical solution.

SOME EXAMPLES AND APPLICATIONS

In the previous section, we have explained in a quite academic form a method to analyze an optical system by using ray tracing. Now we are going to apply this method to several examples. These examples clarify the use of the virtual or real character of the objects and images, virtual and real trajectories, and some other assumed-to-be-known properties and behaviors that are not trivial in the very first attempts to analyze them.

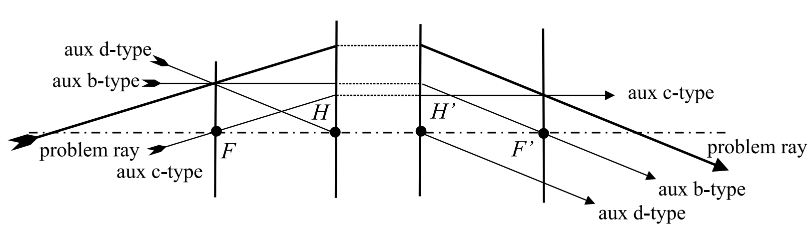

Fig. 3 Graphical solution for a given problem ray using several types of auxiliary rays in a system immersed in air (the nodal points coincide with the principal points).

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Spherical Diopter

Let us assume that we are analyzing the image-forming properties of the cornea and we are interested in the location of the image of a real object placed on the optical axis, in front of the eye at 12 mm from the corneal vertex (Fig. 4). This optical system can be modeled as a spherical diopter of radius 8 mm separating two media of indices $n=1$ (air) and $n'=1.34$ (cornea). First of all, we fix the optical axis as a line containing the center of curvature of this diopter. After some basic calculation, we obtain the values of the focal distances, $f=-23.5$ mm and $f'=31.5$ mm. These distances are measured from the object and image principal planes, respectively. After applying the principal planes definition, we can establish that the system is thin and their principal plane coincides with the intersection of the optical axis with the corneal vertex. Then, the first step allows to obtain a graphical layout of the system containing the location of the principal planes, H and H', and the focal points and planes, F and F'. For a spherical diopter, the nodal points, N and N', are located at the center of curvature of the diopter. In the second step, we define a strategy to locate the image. The object is on the optical axis. Therefore the optical axis ray (a-type) can be used to conclude that the image will be placed on the optical axis also. Then, just by tracing another ray departing from the object, we will obtain the image at the point where this ray intersects the optical axis. The third step is the ray tracing itself. None of the ''easy'' rays but the optical axis ray can be directly used in this problem. Therefore we may choose an arbitrary ray that passes through the object point and arrives to the object principal plane. As the image principal plane coincides with the object one, the ray in the image space will depart from the same point of arrival of this ray at the principal plane location. To properly trace this ray, we use another auxiliary ray in the object space. In the figure, we have traced the three possible auxiliary rays of types b, c, and d. It should be noted that for tracing the incoming parallel-to-the-axis auxiliary ray (b-type), we have elongated the problem ray backward to reach the object focal plane. The problem ray in the image space is a ray whose real trajectory does not intersect the optical axis. However, the image can be obtained by elongating this real trajectory on the left side of the diopter. This is the virtual portion of this ray, but it is still the same ray in the image space. Then, the image is virtual and located at the intersection of this virtual trajectory with the optical axis. We should remind that the image space is never restricted to the semispace to the right of the image principal plane (neither the object space is restricted to the semispace to the left of the object principal plane). The image space is wherever the images are formed (real or virtually), and the object space is wherever the objects are located (real or virtually). The fourth step is the translation of the graphical results in an analytical form. If the plotting has been made faithful in distances and locations, we should be able to measure the image distance from any reference point. The image has been obtained using a virtual trajectory and it will be virtual itself. Then, when observing behind the cornea, the rays coming from the object will apparently be coming from the obtained image located farther than the object. A double check of these results could be performed by using the paraxial optics formula.

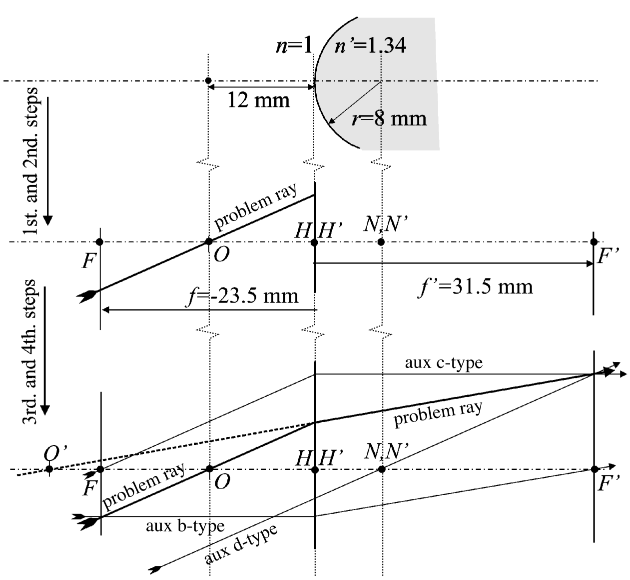

Fig. 4 Ray tracing to find the location of the image given by a spherical diopter. After obtaining the location of the principal, focal, and nodal points, the ray tracing of a given ray is made using auxiliary rays. The image is virtual.

The Thin Lens

Let us take a thin lens of positive focal, $f'=100$ mm. In front of the lens, we place an object with transversal size of 10 mm and located at 300 mm from the lens (see the top of Fig. 5). The problem is to obtain the position and size of the image. In this case, the optical parameters are explicit and the first step of our method is quite easy. If the lens is thin and positive, its representation is a segment terminated with outward-pointing arrows at both sides (inward-pointing arrows for negative thin lenses), located perpendicular to the optical axis. The object will be placed at the given distance and it is usually represented by another arrow that finishes at the height given in the problem. Sometimes, when translating faithfully the data into the paper, the lateral size of the object is very small and it jeopardizes a clear representation and ray tracing. By using the previously explained extension of the Lange

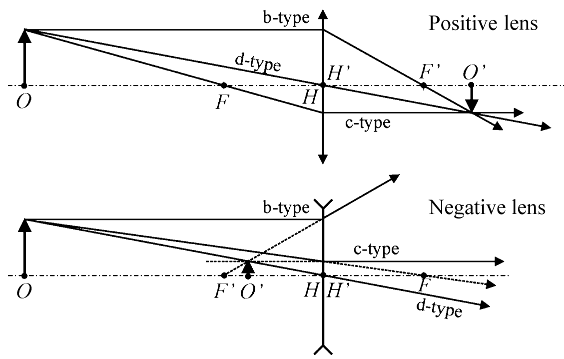

Fig. 5 Ray tracing to find the image location and size, through a thin lens. The drawing in the top is for a positive lens and produces a real image. The plot at the bottom is for a negative lens and produces a virtual image.

formula, we have license to expand the lateral size and use larger angles for the rays to be traced. This is why in this first step we have enlarged the lateral size of the object to make it manageable. The second step is to realize that by tracing a couple of rays from the extreme of the object, it is possible to obtain the image of this extreme reconstructing the whole image. The third step is the ray tracing itself. In the figure, we have traced three ''easy'' rays of types b, c, and d, departing from the extreme of the object, although only two of them are necessary to solve the problem. The extreme of the image is obtained and the rest of the object is extended until the optical axis on the image plane. The fourth step is the analysis of the results. In this case, the image is inverted with respect to its original orientation, is real, and its size is smaller than the object.

If we change the sign of the focal length of the lens maintaining the object location unchanged, the results are quite different (see the bottom of Fig. 5). However, the first, second, and third steps are the same and we only need to adapt the ray tracing to the negative character of the lens. This means to use virtual trajectories to find the location of the image. Therefore when applying the fourth step, we have to conclude that the image is virtual, its orientation has not been changed, and the lateral magnification is lower than 1.

A Compound System: The Microscope

A microscope is formed by two main optical subsystems: the objective and the ocular. The objective is typically a high-power optical system characterized by its numerical aperture and its magnification. The numerical aperture is of limited use in the paraxial regime, but the magnification allows us to obtain the focal length. The magnification is also a characteristic parameter of the ocular subsystem. Let us assume that we have a microscope formed by an objective of $20\times$ and an ocular of $10\times$. The length of the tube of the microscope is $t=160$ mm (this is the distance between the image focal point of the objective and the object focal point of the ocular). The ocular and the objective are compound elements for which their object and image principal planes do not coincide. In this example, the separation between the principal planes in the objective is 10 mm and in the ocular is 15 mm. Our goal is to locate the focal and principal planes of this compound system (Fig. 6). To apply the first step, we calculate the focal distances of the objective and the ocular by using the definitions of magnification for the objective and the ocular of a microscope. The results are $f'_{\mathrm{obj}}=8$ mm and $f'_{\mathrm{ocu}}=25$ mm. The second step uses the definitions of the image and object focal points as the conjugated points of the infinity in the object and image space, respectively. Then, in the third step, we will trace an incoming parallel-to-the axis ray to find the image focal point. This b-type ray needs an auxiliary ray to progress through the ocular. The object focal point is obtained tracing a c-type ''easy'' ray backward through the microscope. Again, an auxiliary ray will be necessary to cross the objective toward the object space. The fourth step analyzes the results. The intersections of these rays with the optical axis are the focal points. To obtain the principal points, we elongated the portions of these rays in

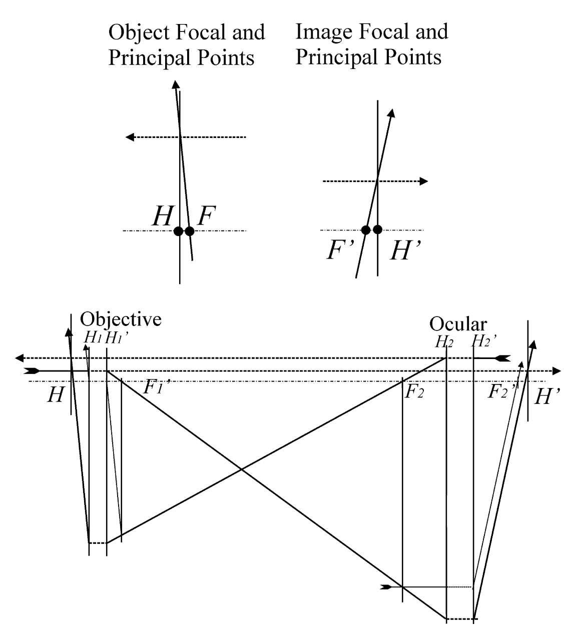

Fig. 6 Ray tracing to find the focal and principal plane of the whole microscope. We can see that the angles involved in the ray tracing are well beyond the paraxial approach; however, they are still valid. We have detailed how to find the focal and principal planes in the insets.

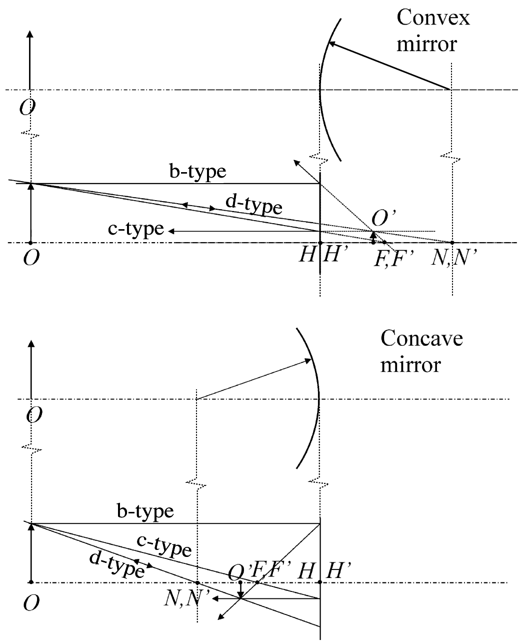

Fig. 7 Ray tracing for two mirrors having the same value of the modulus of the radius of curvature, but different signs. We have made use of the ''easy'' rays to find the image.

the object and image space. The intersection given by the ray defining the image focal point belongs to the image principal plane. For the object principal plane, we proceed accordingly.

Mirrors

The ray tracing in mirrors follows exactly the same rules than in dioptric elements. It only changes the direction of the propagation of light. An interesting property of spherical mirrors is that the image focal point and the object focal point coincide in the same position. This position is located at the midpoint between the center and the vertex of the mirror. As in the case of spherical diopters, a ray passing through the center of curvature of the surface does not change its angle with respect to the axis. Then, nodal points are located at the center of curvature.

In Fig. 7, we have obtained the location and size of a real object for two mirrors differing only in the sign of their curvature; one of them is concave and the other is convex.

Apertures, Field of View, Pupils, and Windows

Besides the intuitive insight obtained from paraxial ray tracing in the analysis of optical systems, the evaluation of the aperture and field of view is probably one of the most interesting applications of ray tracing. The lateral limitations can be analyzed easily with this method. A typical application is the location and size of an optimum stop to properly limit the field of view. The graphical language used in ray tracing makes easier to know which is the diaphragm of aperture in a compound system, if the stop has to be enlarged, diminished, or moved along the axis, if some other lens need adjustment in lateral size, etc.

As an example, we analyze the case of a compound system of two positive thin lenses having focal lengths of $f_1'=30$ mm and $f_2'=80$ mm and separated a distance of 90 mm. The object plane is located in front of the first lens at a distance of 120 mm. The goal is to give the dimensions of the lenses and the location of any necessary stop to make a system having an F# of 4 for the object at infinity and a field of view comprising a circle in the object plane of 40 mm in diameter with no vignetting. The calculation of the focal of the compound system yields $f'=120$ mm. Then, to have an F# 1:4, the diameter of the entrance pupil has to be 30 mm. If we want the first lens to be the diaphragm of aperture, it will be the entrance pupil also. Therefore the F# of the first lens is 1, and its diameter is 30 mm. To comply with this specification, we

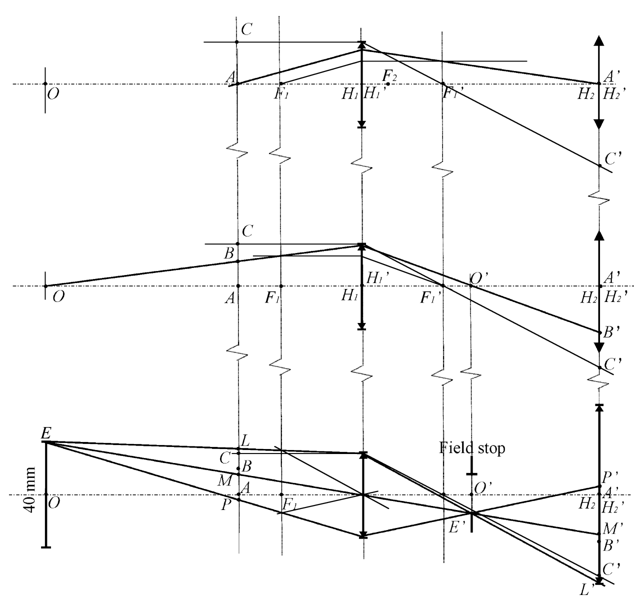

Fig. 8 The plot at the top of the figure shows the location of the second lens in the object space. This location is used to find the minimum size of the second lens to have the first lens as the diaphragm of aperture. The central ray tracing also finds the location of the image, O', after crossing the first lens. The bottom ray tracing shows the field rays departing from the extreme of the object and reaching the second lens. From this last ray tracing, we can see that the minimum size of the second lens is given by the point L'. At the same time, to avoid vignetting, a field stop is placed at the location of the intermediate image. Its diameter is also given from the ray tracing.

trace rays, in backward propagation, to locate the position of the second lens in the object space (see the ray tracing in the top of Fig. 8). This ray tracing produces conjugated points *A*–*A*′ and *C*–*C*′. The location of *C*′ provides a minimum size of the second lens to have the first lens as the diaphragm of aperture for the infinity. Now to solve the dimensions of the second lens, and the location and size of an intermediate stop, we trace more rays through the system (ray tracing in the middle of Fig. 8). The first one will be departing the object plane at its intersection with the optical axis, *O*, and will pass through the outer limit of the first lens. Its ray tracing through the system is possible by using auxiliary rays when necessary. When analyzing the results, we see that there exists a location between both lenses where a real image, *O*′, is formed. This will be a candidate for placing an intermediate stop. The intersection of this ray with the second lens, *B*′, provides a limitation of the size of this lens. If the second lens were smaller than this limit, then the diaphragm of aperture for this object position would be this second lens. The other parameter of the optical system is the expected field of view. This field of view is measured from the center of the entrance pupil of the system that we have assumed to be the rim or frame of the first lens. A field ray passing through the center of the entrance pupil is traced from the limit specified for the field in the object plane, point *E* (this ray passes through points *M*–*M*′ in Fig. 8). Two other field rays are traced from the same point of the object plane and passing through the extreme points of the entry pupil. The real trajectories in between the lenses of these three rays intersect at a given point on the intermediate image plane and they travel to the second lens. The three rays intersect the second lens at different heights, *P*′, *M*′, and *L*′. The fourth step of this ray tracing makes possible to conclude a couple of things. First, the optimum location of the intermediate stop is at the position where the intermediate image is found. This location is necessary, although not sufficient, to preclude vignetting. The size of the intermediate stop is given by the specified field of view and should not be smaller than the size of the intermediate image. Second, the size of the second lens should allow the three field rays to go through it. If any of the rays were stopped, then the final image would be vignetted by the second lens. If these two conditions are fulfilled, the field stop is the diaphragm located in between both lenses. Further ray tracing may be used to find the size and location of the exit pupil and the entry and exit windows.

CONCLUSION

In this article, we have revisited the main concepts and characterizing parameters of optical systems within the paraxial approach. This has been performed by using a few, but consistent, rules based on paraxial optics. Then, paraxial ray tracing has shown unique capabilities to clarify the behavior of light through simple and compound systems. In addition, the limitations of paraxial ray tracing have been analyzed. Several examples, showing the application of paraxial ray tracing to actual problems, have been used to clarify the method.

REFERENCES

1. Rolland, J.P.; Shaoulov, V.; González, F.J. The art of back-of-the-envelope paraxial ray-tracing. IEEE Trans. Ed. **2001**, *44* (4), 365–372.
2. Ghatak, A.K.; Thyagarajan, K. Paraxial Ray Optics. In *Contemporary Optics*; Plenum Press: New York, 1978; 1–30.
3. Mouroulis, P.; Macdonald, J. Gaussian Optics. In *Geometrical Optics and Optical Design*; Oxford University Press: New York, 1997. Chapter 4.2.
4. Boreman, G.D. Geometrical Optics. In *Basic Electro-Optics for Electrical Engineers*; SPIE Optical Engineering Press: Bellingham, WA, 1998; 1–22.
5. Hecht, E. *Optics,* 3rd Ed.; Addison Wesley: Massachusetts, 1998.
6. Smith, W.J. *Modern Optical Engineering,* 3rd Ed.; McGraw-Hill: New York, 2000.
7. Alda, J. *Paraxial Optics, in Encyclopedia of Optical Engineering*; Driggers, R., Ed.; Marcel-Dekker: New York, 2003; 1920–1931.