Contact angle calculation for sessile drops with data obtained from top-view measurements

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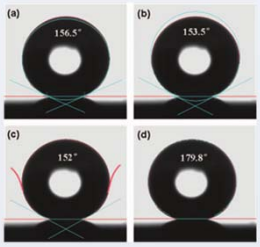
INTRODUCTION

The bibliography presents multiple, easy and direct ways to calculate the contact angle from a side-view image of a sessile drop [1-3]. However, if one wants to measure the drop with a device that only allows viewing the drop from the top, there are few direct options to perform the same calculation [4,5].



Images of the same water droplet under different fitting modes of the static contact angle (from [3])

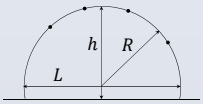
This work presents several alternatives for obtaining the contact angle in a top-view configuration. As input data, these mathematical methods require several parameters of the drop that can be obtained with a non-contact profilometer.

Broadly speaking, non-contact profilometry is used to measure a surface's profile. There are a wide range of techniques which are currently being employed, such as laser triangulation, confocal microscopy, low coherence interferometry, digital holography, etc. By means of these techniques several parameters of the drop, such as its height and its apparent diameter, the coordinates of its apex, etc, can be measured.

DISCUSSION

Four different method are presented for the calculation of the contact angle θ of a liquid sessile drop with information obtained in a top-view configuration. Each method uses an adequate set of parameters which can be measured with a non-contact profilometer:

- Height h,
- Apparent diameter L or
- Coordinates of different surface points from the top of the drop



The volume of the drop is known and must be small enough to discard gravity effects, so the shape of the drop can be approximated with a truncated sphere.

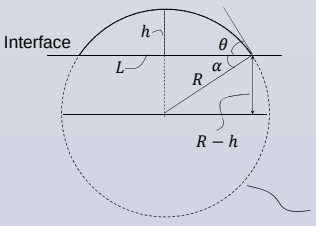
- **First method**

Measured parameters: h, L

Description: This method is based on a purely geometric calculation of the radius R of the drop using its height $\boldsymbol{h}$ and its apparent diameter $\boldsymbol{L}$. With this information, it is possible to calculate subsequently the contact angle θ. By this method, the volume of the drop is not involved.

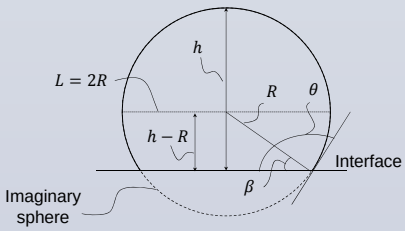
Depending on the wettability properties of the surface, two situations must be taken into account:

- Case a: $\theta < 90°$ and $h < L/2$



$$\theta = \mathrm{acos}\left(\frac{\frac{L^2}{4} - h^2}{h^2 + \frac{L^2}{4}}\right)$$

- Case b: $\theta > 90°$ and $h > L/2$



$$\theta = \mathrm{acos}\left(\frac{L - 2h}{L}\right)$$

- **Second method**

Measured parameters: V, h

Description: this method uses the formula that relates the volume $\boldsymbol{V}$ of a spherical cap to its height $\boldsymbol{h}$ and radius R to calculate the drop's radius R.

$$V = \frac{\pi h^2}{3}(3R - h) \quad \rightarrow \quad R = \frac{1}{3}\left(\frac{3V}{\pi h^2} + h\right)$$

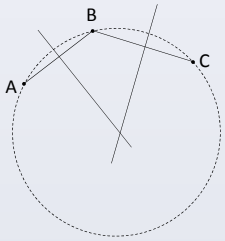
- Contact angle calculation

$$\theta = \mathrm{acos}\left(\frac{3V - 2\pi h^3}{3V + \pi h^3}\right)$$

- **Third method**

Measured parameters: the coordinates of at least three different points located along a meridian of the spherical cap are measured by non-contact profilometry.

Description: The drop's radius is measured by the perpendicular bisector method.



- Each point is represented as $P = (x_i, z_i)$
- The expression for the perpendicular line between two points is obtained:

$$z - z_{ij} = M_{\perp ij}(x - x_{ij})$$

Where
- (x, z) is the origin of the coordinates system
- (x_{ij}, z_{ij}) is the middle point defined by two points
- $M_{\perp ij}$ is the slope of the perpendicular segment defined by these two points

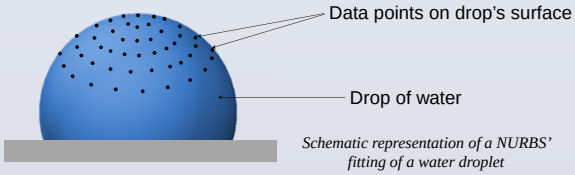
- The radius is then obtained:

$$R = \sqrt{(x_i - x)^2 + (z_i - z)^2}$$

- **Fourth method**

Measured parameters: a set of data points on the drop's surface which are obtained by non-contact profilometry.

Description: this method uses a fitting with a None Uniform Rational B-Splines (NURBS) surface of a spherical cap to fit this set of data points to obtain the value of the drop's radius, and subsequently the contact angle can be calculated.



Schematic representation of a NURBS' fitting of a water droplet

CONCLUSIONS

This work addresses the problem of obtaining the contact angle of a sessile drop from top-view measurements. It presents four mathematical methods which are adequate for modeling any sessile drop with contact angles between 0° and 180°, using different parameters of the drop which can easily be measured with a non-contact profilometer.

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