Strategy to obtain initial configurations for free form reflectors design

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ABSTRACT

A strategy to obtain sets of initial configuration to design freeform reflector surfaces is presented. This strategy brings the initial configuration of the reflector surface using a collection of elemental facets defined by Bezier surfaces and able to face the optimization process of the illumination system.

The purpose of this communication is to provide initial configurations to obtain a set of parameters defining the freeform facets described by Bezier curves. Those parameters can be modified by a global optimization process of the lighting system. This task can be accomplished using a set of simple geometric elements that are the basis for calculating a first approximation to the facet surface. The proposed strategy provides a simple geometric design method to perform valid initial configurations for lighting systems with reflective surfaces that can be further optimized.

The method to calculate the geometry of every single facet is based on ray tracing and uses a merit function to find the parameters defining the Bezier curve that best meets specifications in each elementary facet.

Applying this method to 2D tangential and sagital axes, a network of control points are obtained for describing a Bezier surface compatible with any standard optical optimization tool and suitable for viewing with CAD tools.

Keywords: non imaging optics, optical design freeform, reflector, faceted mirror

1. INTRODUCTION

An optical design process is defined by a merit function, a quality criteria and an optimization algorithm. These are the basic tools to lead the optical system to the final required conditions[1].

It is common knowledge among designers that, if the optimization process starts from a bad initial configuration, the system requires long time to achieve a satisfactory result and the design success is not always guaranteed. Having a quick and simple strategy able to construct a set of valid initial conditions could be a powerful tool on the optimization processes, specially on those requiring a big initial population, as methods based on genetic algorithms[2].

Reflector design process follows a completely different path than classical optical design. It is mainly based on energy contribution and its spatial and uniform distribution, thus reflector designs are supported on the relationship among three optical design basic elements: source, mirror and target[3,4]. This relationship will establish the geometrical relations among the elements, and also will lead to the criteria to reach the appropriate merit function.

On a reflector design process three essential steps are key point in order to obtain the final reflector geometry :

-Segmentation process that leads to mirror facets. Source and target segmentation are associated to the mirror segmentation process and it must be accompanied by an algorithm linking the segments of each three optical design basic elements .
- Position and orientation of every single facet.
- Facet local geometry in order to obtain the final goal of energetic distribution on the target.

The benefits of using segmented optical surfaces in non imaging optics is widely documented on literature [3 4 5 6 7 8]. The need of faceting the mirror is for locally adjust the geometric parameters of the facets, thus the illumination goal at the target could be reached.

Different segmentation criterion might be defined for each of the three elements to accomplish different energetic distribution criteria. It must be taken into account that initial conditions could be on a wide range of possibilities, as near of far target for instance, thus the criteria must be flexible in order to adapt to any situation. Source segmentation criteria, is, in general defined by the geometry and type of the source, while target segmentation is defined by the final goals of the illumination system.

In this way, a process to build faceted optical surfaces from a geometrical structure is defined. There are several segmentation methods described by different authors[9 10] but on all of them, two criterion must be taken into account:

- Each facet must contribute on a similar way to the final result.
- Size variation between consecutive facets must be smooth.

These two criterion lead to a segmentation process based on energetic distribution more than purely geometric criteria (fgure 1). A relationship algorithm linking source, mirror and target segment must be also defined

a b

Figure 1: Example of segmentation methods. Segmentation method based on concentric division a) Geometric criteria: equiangular division b) energetic criteria :segments of equal area based on golden ratio

Once the segmentation criterion are set, depending on the design environment conditions, the facet position in terms of pitch and tilt, must be described . The spatial orientation must be set in terms that, on the first approach of a principal ray, the facet achieves the defined relationship between source, mirror and target.

However the aim of this paper is to get initial shape configurations for the elementary surfaces, so we avoid the discussion about surfaces' size and segmentation.

The third step and the main objective of this contribution is to completely define the geometry of each facet, that is, to describe each curvature in order that the illumination on the target achieves the required conditions. In literature, we find mainly geometries based on conics surfaces[11]. This has the advantage of simplicity but introduces a difficulty when the complete mirror has to be described mathematically in order to be manufactured, so we characterize the facets local geometry by Bezier curves. Afterwards the local geometry can be modified by a global optimization process of the lighting system.

2. OBJECTIVE

The objective of the communication is to set the geometrical parameters of the individual mirror's freeform facets as initial conditions for further optimization. The freeform facets are described by means of Bezier surfaces that can be modified by a global optimization process[2].Dealing with Bezier surfaces instead of conical surfaces has the advantage that can be represented and modified with commercial CAD tools because Bezier surfaces are easily transcribed as B-Splines or NURBS [12 13]. Once the surface is finally optimized, if defined by the standard Bspline parameters, it can be automatically transcribed to manufacturing tools in order to build them, being a fundamental tool for quick implementation into any engineering process.

The facets are treated as singulars and are defined on an individual way but under global conditions. These conditions come from the segmentation process and from the algorisms linking source, mirror and target segments described on previous steps.

3. PROCEDURE

The freeform facet Bezier surface to be defined is a 3x3 bicubic tensorial surface with 4x4 control points. A method is defined to find the surface's control points. It is a constructive method based on 2D Bezier curves calculated on strategic planes previously selected. The Bezier curves are defined under optical and energetic criterion.

In order to calculate the facet's local geometry ,the process starts from a flat reflector surface which outline is previously set on the mirror segmentation process. The proposed calculation method allows to transform a flat facet to a new Bezier surface that accomplish the optical features described by the algorisms linking source, mirror and target. In order to generate the Bezier surface, target and source are considered both as point objects, and the four corners of the flat starting facet will be the four corners of the new Bezier surface.

A Bezier surface can be obtained from the Bezier curves contained on it, that is, given 4 Bezier curves and its control points, the Bezier surface containing them can be defined by the appropriate combination of control points. Taking into advantage a segmentation of the space enclosed into the solid angle formed by the point source and the facet is done, cutting the solid angle into two horizontal and four vertical slices (figure 2). Each of those slices are considered as strategic planes for calculation purposes. On each of those plans, a 2D algorithm is applied to obtain the best reflecting Bezier curve that gives the maximum concentration of energy around target from rays coming from the source. The obtained curves are 3rd order Bezier curves, that is, with 4 control points. With the six Bezier curves obtained, one for each defined slice, a mesh of control points is obtained, that combined properly, define a univocal Bezier surface containing those defined Bezier curves. In this way a surface is described by means of CAD compatible parameters, thus the surfaces can be easily introduced on optical software to optimize the system or on mechanical software to build it up.

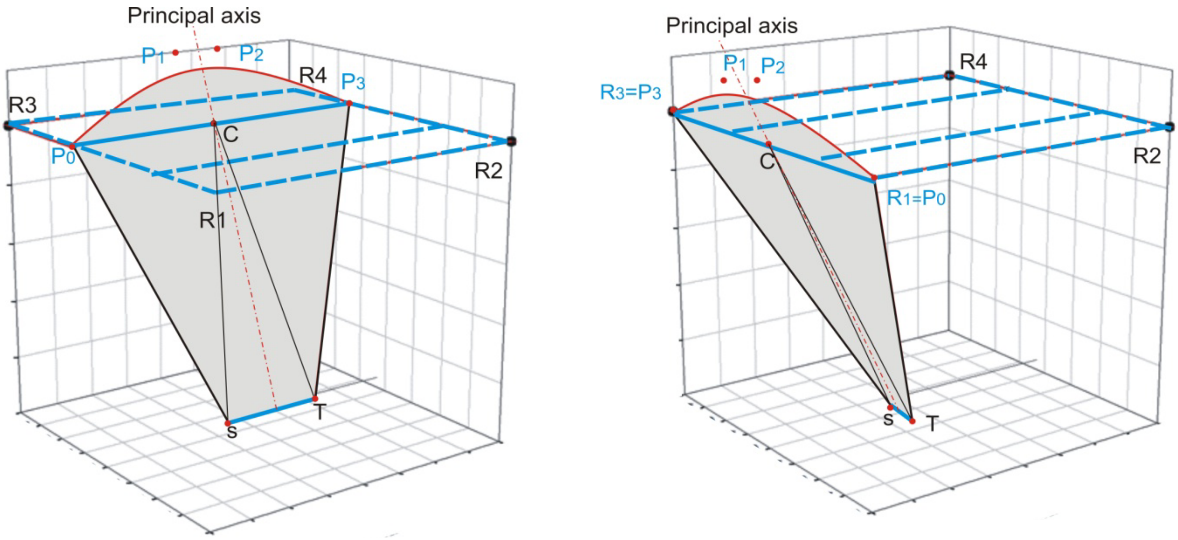


Figure 2: Position of point source S, point target T and the initial flat surface outlined by points R1, R2, R3 y R4. Vertical and horizontal axis are showed in blue lines. The continuous blue line is one of the principal horizontal axis in which a plane is defined and the 2D algorithm is applied in order to find the best Bezier curve. P_0 and P_3 are the ends of the Bezier curve, while P_1 and P_2 are the two control points that are calculated by using he proposed 2D algorithm.

3.1 Algorithm on 2D

On the following sections, the steps to obtain the best reflecting Bezier curve that gives the maximum concentration of energy around target from rays coming from the source are explained.

3.1.1 Ray tracing through Bezier curves

The first step is to develop the 2D ray tracing algorism through Bezier curves that follow equation (1)

$$B(t) = (1-t)^3\, P_0 + 3(1-t)^2\, tP_1 + 3(1-t)t^2\, P_2 + t^3\, P_3. \tag{1}$$

where P_0, P_1, P_2, P_3 are the control points of the curve an t the parameter to go along through the curve. Being the equation of a Bezier curve a continuous function in all points, the derivative can be found as

$$B'(t) = 3t^2\, C_1 + 2tC_2 + C_3 \tag{2}$$

Where the C_1 , C_2 , C_3 are the following coefficients

$$C_1 = (-P_0 + 3P_1 - 3P_2 + P_3) \quad C_2 = 3P_0 - 6P_1 + 3P_2 \quad C_3 = -3P_0 + 3P_1 \;\; D = P_0 \tag{3}$$

so the tangent line on any point of the curve can be found. Thus, given a point source, a fan of rays covering a segment of the Bezier curve is defined, the impact point Q of the ray on the Bezier curve is calculated and consequently a direction $\vec{R}$ for each ray is obtained (figure 3) . On the other hand, the derivative of the surface on each impact point allows to calculate the direction of the ray with respect the normal direction $\vec{N}$ on the curve. The incident angle θ is calculated as follows

$$\sin\theta = \frac{|\vec{R} \wedge \vec{N}|}{|\vec{R}||\vec{N}|} \tag{4}$$

The reflecting direction θ' is calculated using the Snell law in mirrors, $sin\theta = -\sin\theta'$. Then a rotation of θ' applied to $\vec{N}$ gives the direction of the reflected ray $\vec{R'}$. If the target is defined by a point and a direction forming a line, a collection of impact points coming from the reflected rays can be calculated.

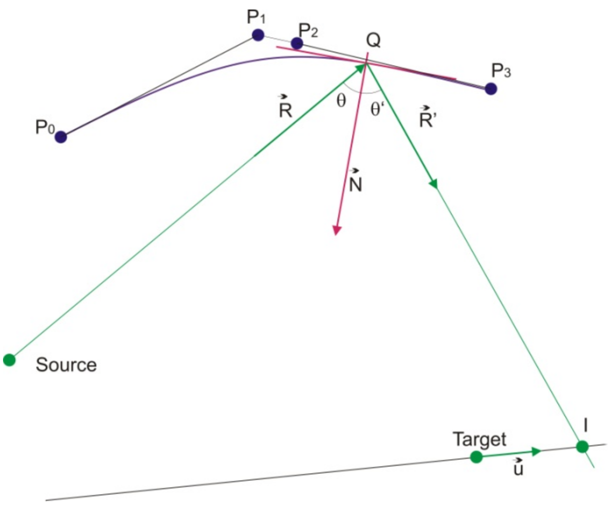


Figure 3: Snell law applied to a Bezier curve. Scheme of calculated values: impact point Q, derivative of the curve in Q, $\vec{N}$ as the normal direction of the curve in Q, incident angle θ, reflecting angle θ', direction of the reflected ray $\overrightarrow{R'}$ and impact point I

3.1.2 Design strategy in 2D

Having set-up the basis of ray tracing through Bezier curves, the initial conditions design strategy can be defined. The start up point are four points laying in the strategic plane: source point (S), target point (T) and the two extreme points contained on the outline of the facet. The later points are defined as the end points of the desired Bezier curve (P_0, P_3). The goal is to find the two remaining Bezier control points (P_1, P_2) in order to obtain the best Bezier curve for the defined reflecting purposes.

Prior to define the strategy, some parameters must be defined

- **Principal axis** is the bisector of the lines SC and ST , being C the central point of the segment $\overline{P_0P_3}$.
- **Shape parameter *L*** is related to the local curvature degree of the Bezier curve. A high value of the shape parameter L leads to plane curves, whilst a low value of the shape parameter leads to more closed curvatures.
- **Progress factor *K*** is a variable in the local optimization process.

A perpendicular segment to the principal axis and centred on *C* is set up. The length of the segment is fixed by the shape parameter *L* and it is discretely moved through the axis following the progress factor *K*. In each progress step the points P_1 and P_2 are set as the ends of the segment, thus all parameters required to define a Bezier curve are obtained. (Figure 4). Moving the segment along the principal axis, a collection of Bezier curves is obtained

A fan of rays from the point source S is pointed towards each defined Bezier curve, so the reflecting rays and the impact diagram around the target is obtained for every single Bezier. Thus, using a weighted merit function of vicinity of the impact points with respect the point target T, the better Bezier curve is selected.

The merit function used is

$$MF = \frac{N}{\varepsilon} - \frac{N}{\sum_{n=1}^{N} D(n) + \varepsilon} \tag{5}$$

where N is the number of rays used, *D(n)* is the distance from every impact point to the target point T and and ε is a security factor to avoid singularities on the MF

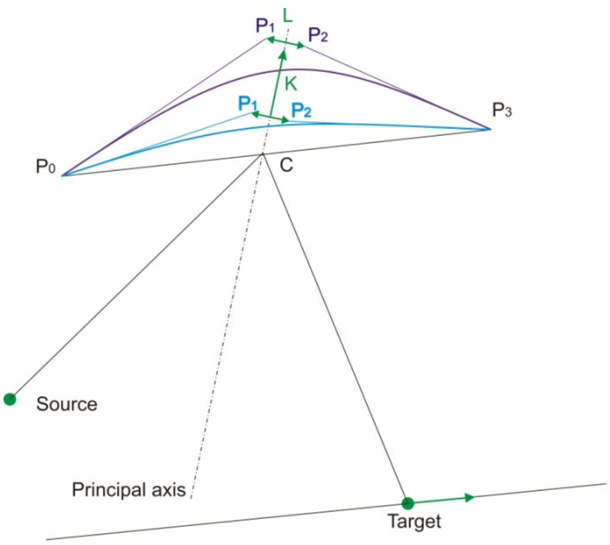


Figure 4: Two different curves obtained by displacing control points P_1 and P_2 along principal axis by a progress factor K . Distance between P_1 and P_2 is set by shape parameter L.

3.2. From curves to surfaces , algorithm on 3D

3.2.1. Description of bicubic 3x3 tensor product Bezier surface

In this section some facts about Bezier surfaces are presented in order to better understanding the reflective surfaces constructive method

The equation of a 3x3 Bezier surface is

$$P(s,t) = \sum_{i=0}^{3}\sum_{j=0}^{3} P_{ij} B_i^3(s)\, B_j^3(t) = \sum_{i=0}^{3}\left(\sum_{j=0}^{3} P_{ij} B_i^3(s)\right) B_j^3(t) \tag{6}$$

Where P_{ij} are the control surface points , P(s,t) are points of the surface, *s* and *t* are the parameters of the surface, and $B_i^3(s)$ and $B_j^3(t)$ are Bernstein polynomials which general formula is

$$B_i^m(s) = \binom{m}{i}(1-s)^{m-i}s^i \tag{7}$$

The control net of a 3x3 Bezier surface is a 4x4 control points mesh, thus the control points for a bi-cubic facet can be organized as follows

$$P = \begin{bmatrix} P_{03} & P_{13} & P_{23} & P_{33} \\ P_{02} & P_{12} & P_{22} & P_{32} \\ P_{01} & P_{11} & P_{21} & P_{31} \\ P_{00} & P_{10} & P_{20} & P_{30} \end{bmatrix} \tag{8}$$

The corresponding blending functions to express the surfaces in *s* and *t* parameters are

$$\begin{bmatrix} (1-s)^3t^3 & 3s(1-s)^2t^3 & 3s^2(1-s)t^3 & s^3t^3 \\ (1-s)^3 3t^2(1-t) & 3s(1-s)^2 3t^2(1-t) & 3s^2(1-s)3t^2(1-t) & s^3 3t^2(1-t) \\ (1-s)^3 3t(1-t)^2 & 3s(1-s)^2 3t(1-t)^2 & 3s^2(1-s)3t(1-t)^2 & s^3 3t(1-t)^2 \\ (1-s)^3(1-t)^3 & 3s(1-s)^2(1-t)^3 & 3s^2(1-s)(1-t)^3 & s^3(1-t)^3 \end{bmatrix} \tag{9}$$

where each row and column can be seen as an iso-parametric Bezier curve. The whole surface can be thought as a collection of iso-parametric curves and the construction of such curves is described in terms of **control curves**. Control curves are auxiliary curves that do not lie on the surface. They consist of control points of the iso-parametric curves of *equation 9* and their own control points are surface control points P_{ij} (*equation 3*). Thus, if one of the parameters is fixed, for instance $s=c$, the surface can be expressed as

$$P(c,t) = \sum_{j=0}^{3}\left[\sum_{i=0}^{3} P_{ij}B_i^3(c)\right]B_j^3(t) = \sum_{i=0}^{3} R_j(c)B_j^3(t) \qquad (10)$$

where *R(c)* is the control curve that do not lie on the surface and *P(c,t)* is an isoparametric curve.
The curves defined by control points *Pij ; j = 0; 1; 2;3* (columns) are the control curves steered by parameter *s* , whilst the curves defined by control points *Pij ; i = 0; 1; 2;3* (rows) are the control curves steered by parameter *t*

There are four special iso-parametric curves : the ones defining the surface outline *P(0; t), P(1; t), P(s; 0)* and *P(s; 1).* They are Bezier curves defined by the columns or rows of corresponding control points, thus they are also control curves. In general, the only control points lying on the surface are the four corners (P_{00}, P_{03}, P_{30}, P_{33})

3.2.2 Constructive method to find the Bezier surface control points

Sixteen control points are needed to build up a 3x3 tensor product Bezier surface. In this section, a constructive method to find those control points from several Bezier curves is explained.

The strategic planes are set as slices of the solid angle formed by the point source S and the corners of the facet *(fig 2).* The horizontal direction is fixed as $\overline{ST}$,the parallel direction to the segment between source S and target T. The following six axis will be described in order to find six strategic planes:

- Four axis of the facet contour , two horizontals and two verticals
- Two complementary horizontal axis

In each plane, the described 2D algorithm to find the best Bezier curve is applied. The obtained Bezier curves corresponding to the facet contour are the iso-parametric curves *P(0; t), P(1; t), P(s; 0),* and *P(s; 1)* described on *equation10* (Figure 5b). These curves give the 12 periphery control points depicted in bold in *equation 11* , thus, to obtain the complete mesh of control points the four central underlined control points must be given.

$$P = \begin{bmatrix} \boldsymbol{P_{03}} & \boldsymbol{P_{13}} & \boldsymbol{P_{23}} & \boldsymbol{P_{33}} \\ \boldsymbol{P_{02}} & \underline{P_{12}} & \underline{P_{22}} & \boldsymbol{P_{32}} \\ \boldsymbol{P_{01}} & \underline{P_{11}} & \underline{P_{21}} & \boldsymbol{P_{31}} \\ \boldsymbol{P_{00}} & \boldsymbol{P_{10}} & \boldsymbol{P_{20}} & \boldsymbol{P_{30}} \end{bmatrix} \qquad (11)$$

In order to find the central control points, two complementary horizontal axis are set to obtain two planes were Bezier curves will be find applying the 2D algorithm. The 2D algorithm needs the two end points of the curve to start the process , that is P0 and P3 as stated on equation *(1).* For this purpose, two points of each vertical iso-parametric curves *P(0; t), P(1; t)* are considered. Thus, fixing *t=0.3*, the first and last control point of one of the complementary horizontal axis are P0=*P(0, 0.3)* and P3=*P(1 , 0.3)* respectively .On a similar way , fixing *t=0.6*, the first and last control point of the other complementary axis are P0=*P(0, 0.3)* and P3=*P(1 , 0.3)* respectively. To clarify notation the horizontal parametric curves will be stated as QH and the vertical parametric curves as QV

Six Bezier curves have been obtained. The four horizontal Bezier curves QH have their origin and final points lying on the two vertical Bezier curves QV. Q is the matrix of the control points of the QH iso-parametric curves.

$$Q = \begin{pmatrix} Q_{03} & Q_{13} & Q_{23} & Q_{33} \\ Q_{02} & Q_{12} & Q_{22} & Q_{32} \\ Q_{01} & Q_{11} & Q_{21} & Q_{31} \\ Q_{00} & Q_{10} & Q_{20} & Q_{30} \end{pmatrix} \tag{12}$$

To find the central control points of the surface, only the four horizontal curves QH are necessary taking into account that the peripheral points of the matrix are both categories: surface control points and iso-parametric curve control points.

Isoparametric curves	Bezier curves control points Q_{ij} and the equivalence with surface control points P_{ij}			
QH3 with $t=1$	$\boldsymbol{Q_{03}= P_{03}}$	$\boldsymbol{Q_{13}= P_{13}}$	$\boldsymbol{Q_{23}= P_{23}}$	$\boldsymbol{Q_{33}= P_{30}}$
QH2 with $t=0.6$	$\boldsymbol{Q_{02}= P_{02}}$	Q_{12}	Q_{22}	$\boldsymbol{Q_{32}= P_{31}}$
QH1 with $t=0.3$	$\boldsymbol{Q_{01}= P_{01}}$	Q_{11}	Q_{21}	$\boldsymbol{Q_{31}= P_{32}}$
QH0 with $t=0$	$\boldsymbol{Q_{00}= P_{00}}$	$\boldsymbol{Q_{10}= P_{10}}$	$\boldsymbol{Q_{20}= P_{20}}$	$\boldsymbol{Q_{30}= P_{30}}$

Table 1: Bezier curves control points Q_{ij} and the equivalence with surface control points P_{ij}

The next step is to find the remaining surface control points P_{12} , P_{22} , P_{11} and P_{21} using the Q_{ij} curve control points through the construction of control curves (figure 5c).

As stated on section 3.2.1 the auxiliary control curves are Bezier curves themselves defined by Bernstein polynomials. Taking the second and third control points of QH curves (Q_{1i}, Q_{2i} $i=0,1,2,3$) two control curves can be constructed. These two curves do not lie on the surface, but their control points are surface control points. Taking advantage of this property , the remaining surface control points are calculated (figure 5d).

The general expression of a Bezier surface is

$$P(s,t) = \sum_{i=0}^{3}\sum_{j=0}^{3} P_{ij}B_i^3(s)\,B_j^3(t) = \sum_{i=0}^{3}\left(\sum_{j=0}^{3} P_{ij}B_i^3(s)\right)B_j^3(t) \tag{13}$$

where s and t are parameters of the surface and P_{ij} are surface control points.
The isoparametric curves are calculated freezing s the parameter s , thus for $s=c_k$ values as $c_0=0$, $c_1=0.3$, $c_2=0.6$, $c_3=1$ the coordinates of four curves can be obtained

$$P(c_k,t) = \sum_{i=0}^{3}\left(\sum_{j=0}^{3} P_{ij}B_i^3(c_k)\right)B_j^3(t) \tag{14}$$

On the other hand, four QH Bezier curves calculated via 2D algorithm of section 3.1 can be expressed following the equation

$$P(c_k,t) = \sum_{i=0}^{3} Q_{kj}B_j^3(t) \tag{15}$$

Thus, from equation *14* and *15* , it can be stated the following equality

$$Q_{kj} = \sum_{i=0}^{3} P_{ij}B_j^3(c_k) \tag{16}$$

The Q_{kj} values are known as Bezier control points (table 1) and the specific $B_j^3(c_k)$ Bernstein coefficients can be calculated by equation 7 for $c_0=0$, $c_1=0.3$, $c_2=0.6$, $c_3=1$.

$$B=\begin{pmatrix} B_0^3(0) & B_0^3(0.3) & B_0^3(0.6) & B_0^3(1)\\ B_1^3(0) & B_1^3(0.3) & B_1^3(0.6) & B_1^3(1)\\ B_2^3(0) & B_2^3(0.3) & B_2^3(0.6) & B_2^3(1)\\ B_3^3(0) & B_3^3(0.3) & B_3^3(0.6) & B_3^3(1)\end{pmatrix}=\begin{pmatrix}1 & 0.343 & 0.064 & 0\\ 0 & 0.441 & 0.288 & 0\\ 0 & 0.189 & 0.432 & 0\\ 0 & 0.027 & 0.216 & 1\end{pmatrix} \tag{17}$$

Equaling term by term from equation an*14* and *15* a set of equations is obtained

$$c_0=0\begin{cases} j=0\rightarrow Q_{00}=P_{00}B_0^3(0)=\boldsymbol{P_{00}}\\ j=1\rightarrow Q_{01}=P_{01}B_0^3(0)=\boldsymbol{P_{01}}\\ j=2\rightarrow Q_{02}=P_{02}B_0^3(0)=\boldsymbol{P_{02}}\\ j=3\rightarrow Q_{03}=P_{03}B_0^3(0)=\boldsymbol{P_{03}}\end{cases}$$

$$c_1=0.3\begin{cases} j=0\rightarrow Q_{10}=\boldsymbol{P_{00}}B_0^3(c_1)+\underline{P_{10}}B_1^3(c_1)+\underline{P_{20}}B_2^3(c_1)+\boldsymbol{P_{30}}B_3^3(c_1)\\ j=1\rightarrow Q_{11}=\boldsymbol{P_{01}}B_0^3(c_1)+\underline{P_{11}}B_1^3(c_1)+\underline{P_{21}}B_2^3(c_1)+\boldsymbol{P_{31}}B_3^3(c_1)\\ j=2\rightarrow Q_{12}=\boldsymbol{P_{02}}B_0^3(c_1)+\underline{P_{12}}B_1^3(c_1)+\underline{P_{22}}B_2^3(c_1)+\boldsymbol{P_{32}}B_3^3(c_1)\\ j=3\rightarrow Q_{13}=\boldsymbol{P_{03}}B_0^3(c_1)+\underline{P_{13}}B_1^3(c_1)+\underline{P_{23}}B_2^3(c_1)+\boldsymbol{P_{33}}B_3^3(c_1)\end{cases}$$

$$c_2=0.6\begin{cases} j=0\rightarrow Q_{20}=\boldsymbol{P_{00}}B_0^3(c_2)+\underline{P_{10}}B_1^3(c_2)+\underline{P_{20}}B_2^3(c_2)+\boldsymbol{P_{30}}B_3^3(c_2)\\ j=1\rightarrow Q_{21}=\boldsymbol{P_{01}}B_0^3(c_2)+\underline{P_{11}}B_1^3(c_2)+\underline{P_{21}}B_2^3(c_2)+\boldsymbol{P_{31}}B_3^3(c_2)\\ j=2\rightarrow Q_{22}=\boldsymbol{P_{02}}B_0^3(c_2)+\underline{P_{12}}B_1^3(c_2)+\underline{P_{22}}B_2^3(c_2)+\boldsymbol{P_{32}}B_3^3(c_2)\\ j=3\rightarrow Q_{23}=\boldsymbol{P_{03}}B_0^3(c_2)+\underline{P_{13}}B_1^3(c_2)+\underline{P_{23}}B_2^3(c_2)+\boldsymbol{P_{33}}B_3^3(c_2)\end{cases}$$

$$c_3=1\begin{cases} j=0\rightarrow Q_{30}=P_{30}B_3^3(1)=\boldsymbol{P_{30}}\\ j=1\rightarrow Q_{31}=P_{31}B_3^3(1)=\boldsymbol{P_{31}}\\ j=2\rightarrow Q_{32}=P_{32}B_3^3(1)=\boldsymbol{P_{32}}\\ j=3\rightarrow Q_{33}=P_{33}B_3^3(1)=\boldsymbol{P_{33}}\end{cases}$$

Where the bold P_{ij} points are the required points, whilst the underlined P_{ij} points are known Bezier surface control points This can be expressed as matrix products

$$BS*PS=QS-B_c*P_c \tag{18}$$

Where

$$BS=\begin{pmatrix}B_1^3(c_1) & B_2^3(c_1)\\ B_1^3(c_2) & B_2^3(c_2)\end{pmatrix}\quad PS=\begin{pmatrix}\underline{P_{10}} & \underline{P_{11}} & \underline{P_{12}} & \underline{P_{13}}\\ \underline{P_{20}} & \underline{P_{21}} & \underline{P_{22}} & \underline{P_{23}}\end{pmatrix}\quad QS=\begin{pmatrix}Q_{10} & Q_{11} & Q_{12} & Q_{13}\\ Q_{20} & Q_{21} & Q_{22} & Q_{23}\end{pmatrix}$$
$$B_c=\begin{pmatrix}B_0^3(c_1) & B_3^3(c_1)\\ B_0^3(c_2) & B_3^3(c_2)\end{pmatrix}\quad P_c=\begin{pmatrix}\boldsymbol{P_{00}} & \boldsymbol{P_{01}} & \boldsymbol{P_{02}} & \boldsymbol{P_{03}}\\ \boldsymbol{P_{30}} & \boldsymbol{P_{31}} & \boldsymbol{P_{32}} & \boldsymbol{P_{33}}\end{pmatrix} \tag{19}$$

Thus, Bezier surface control points P_{12}, P_{22}, P_{11} and P_{21} can be calculated solving the following matrix

$$PS=BS^{-1}[QS-B_c*P_c] \tag{20}$$

Obtaining at last, all control points necessaries to build up the surface using equation 13

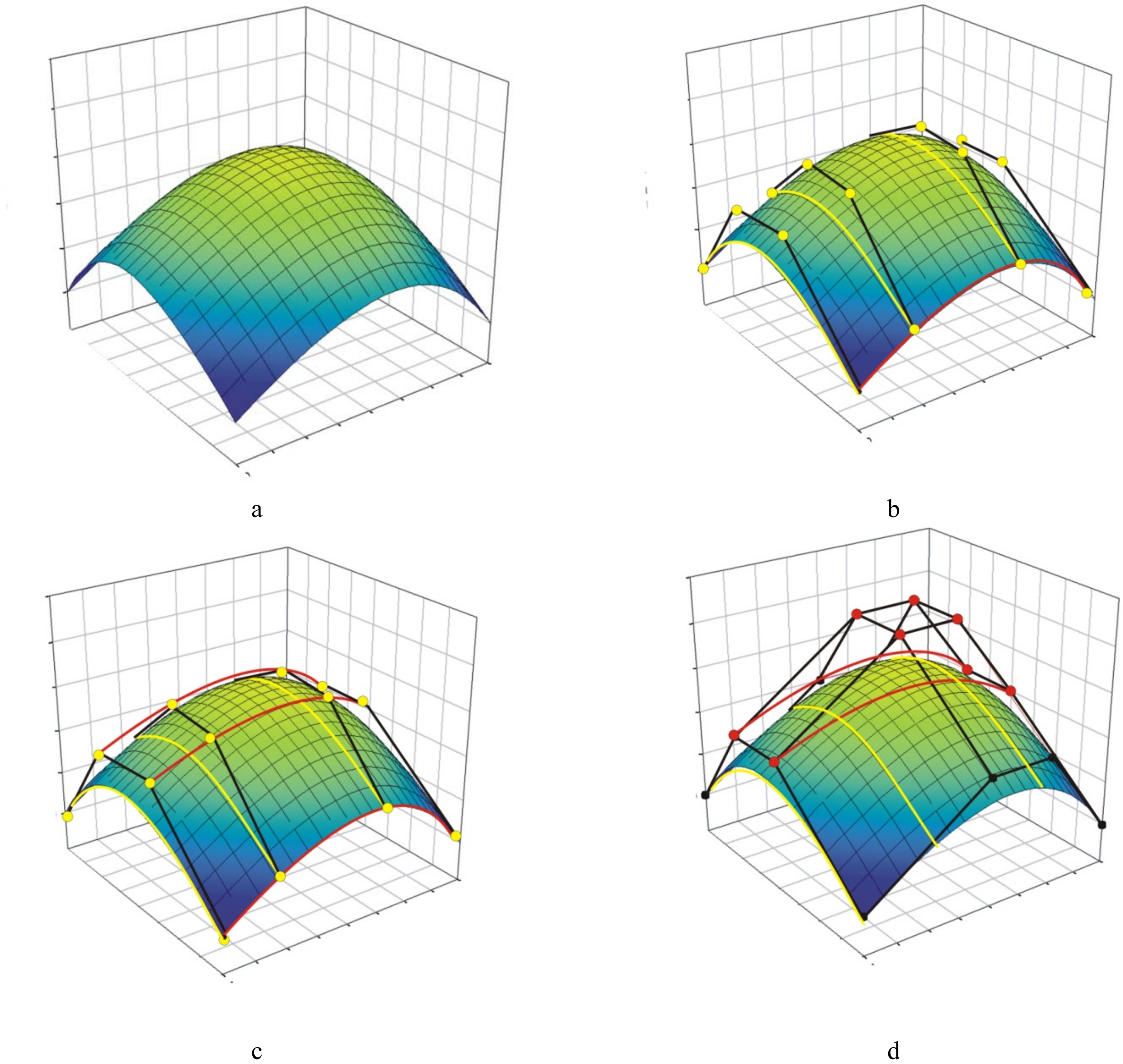


Figure 4 Construction method of a Bezier surface from Bezier curves a) Bezier surface generated by Bernstein polynomials as a tensorial product surface. b) Depicted in red QV vertical isoparametric curves (fixed values: t=0 and t=1). Points of QV curves have been used as ends of QH curves. Control points of QV curves are not represented on the figure. Depicted in yellow four QH horizontal Bezier iso-parametric curves generated with fixed s values. Control points of QH Bezier curves also depicted in yellow. All the curves lie on the surface . c) Depicted in red, two central control curves that do not lie on the surface. Control points of QH Bezier curves lie on control curves. d)The red control points are the control points of the control curves. They are also control points of the surface. The 4 red points of the upper plane are the 4 control point calculated by the equation system.

4. RESULTS

Two examples are presented in order to see the improvement of the final spot diagram applying the proposed calculation method.

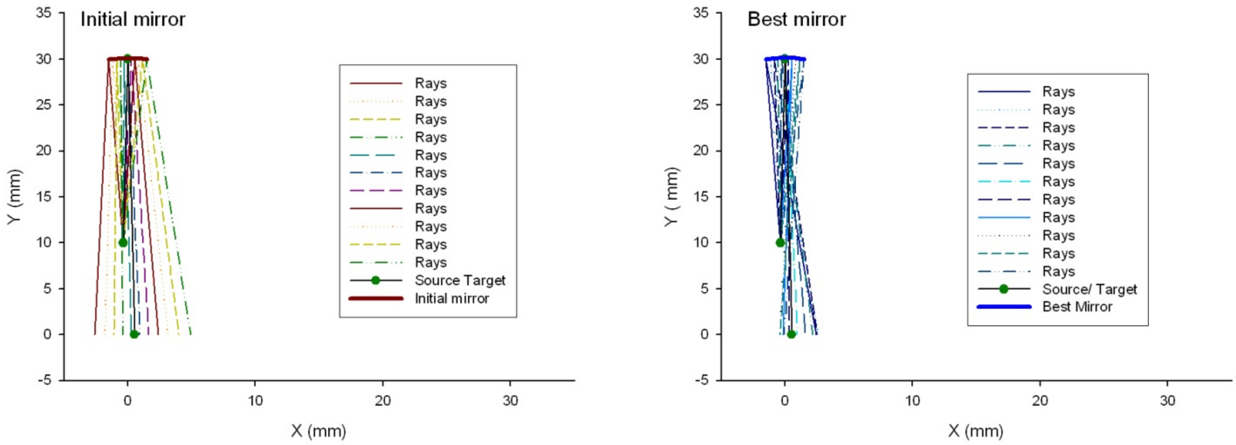
The input data for each example are the four corners of the mirror P_{00},P_{03},P_{30},P_{33}, and the position of source and target

Position of source[-0.349 , 10.,0] Position of target= [0.52 , 0,0] Mirror's dimension 3x3. All dimensions in mm

	P_{0i}	P_{1i}	P_{2i}	P_{3i}
P_{j3}	**(-1.50, 30, -1.5)**	(-1.50,30.10,-0.3)	(-1.50, 30.10, 0.30)	**(-1.50, 30, 1.50)**
P_{j2}	(0.30,30.10,-1.51)	(-0.30, 30.20, -0.30)	(-0.30, 30.20, 0.30)	(-0.30, 30.10, 1.51)
P_{j1}	(0.30, 30.10, -1.51)	(0.30, 30.20, -0.30)	(0.30, 30.20, 0.30)	(0.30, 30.10, 1.51)
P_{j0}	**(1.50, 30, -1.5)**	(1.51, 30.10, -0.3)	(1.51, 30.10, 0.3)	**(1.50, 30, 1.50)**

Table 2 : Best Bezier Surface's control points

Being P_{00},P_{03},P_{30},P_{33} the four original corner points of the mirror



a b

Figure 5 Example A a) : Initial mirror with the reflected rays and spot diagram near the target T. b) Best mirror with the reflected rays and spot diagram near the target T. The spot diagram has been reduced significantly.

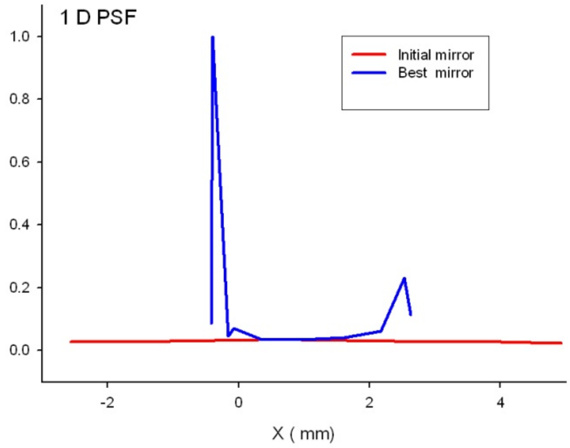


Figure 6 Example A Calcluted 1D PSF by the vicinity of impact points to target. Blue line represents the results obtained with the best mirror found, The length of the spot diagram is significantly smaller than the initial device,

5. CONCLUSION

Within a general method to design freeform reflectors, the conditions to obtain initial configurations to start an optimization process are described. A general outline on how to segment the three essential elements of the design, (source, target and mirror) is described, obtaining position and orientation of mirror's facets. A detailed description on how to obtain the geometry of the facets is presented.

Departing from six points into the space as source, target, and four corners of a mirror, a method is described to find a Bezier surface that best fits with the algorithm linking source, target and mirror. The final result is a collection of Bezier surface control points that can be easily transcribed as B-Splines or NURBS in order to use those with commercial CAD compatible tools

This process can be applied to every facet, thus acting locally facet by facet, a general initial configuration of the mirror is described.

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