A method to measure sub nanometric amplitude displacements based on optical feedback interferometry

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ABSTRACT

Optical feedback interferometry is a well known technique that can be used to build non-contact, cost effective, high resolution sensors. In the case of displacement measurement, different research groups have shown interest in increasing the resolution of the sensors based on this type of interferometry. Such efforts have shown that it is possible to reach better resolutions by introducing external elements such as electro-optic modulators, or by using complex signal processing algorithms. Even though the resolution of the technique has been increased, it is still not possible to characterize displacements with total amplitudes under $\lambda/2$. In this work, we propose a technique capable of measuring true nanometre amplitude displacements based on optical feedback interferometry. The system is composed by two laser diodes which are calibrated within the moderate feedback regime. Both lasers are subjected to a vibration reference and only one of them is aimed to the measurement target. The optical output power signals obtained from the lasers are spatially compared and the displacement information is retrieved. The theory and simulations described further on show that sub-nanometre resolution may be reached for displacements with amplitudes lower than $\lambda/2$. Expected limitations due to the measurement environment will also be discussed in this paper.

Keywords: Optical Feedback Interferometry, High-resolution, Displacement measurement, Nano-metric displacement sensor, Laser diode interferometry.

1. INTRODUCTION

Optical feedback interferometry (OFI), also known as self-mixing interferometry (SMI), is a well known technique that has been extendedly studied during the last three decades. The OFI effect can be defined as a modulation of the optical output power (OOP) of a laser diode (LD) due to backscattering of the beam on a moving target. A formal mathematical model of the phenomena can be found in the classical paper of Lang and Kobayashi[1], where the effects of variables such as the linewidth enhancement factor α, and the feedback factor C are analysed. Even though it is technically possible to generate OFI with any kind of laser, in practice most of the research is done using single mode LDs with built in monitor photodiodes (PD), which permit a direct acquisition of the OOP with relative amplitudes up to -90dB, as it is explained by Donati in Ref 2.

Several practical applications have already been developed for OFI LD based sensors. Most notably, the applications can be grouped by category as follows: displacement sensors[3,4], velocity sensors[5], vibration sensors[6,7], absolute distance measurement[8] and biomedical related sensors[9]. However, in all the cases OFI resolution rested far away from the limits presented by other classical interferometers, where phase shifting method can be easily applied. This difference on resolution limits, reduces the possible number of applications for OFI sensors, thus, making it impractical for new trend industries such as non destructive testing, bio-medical research and new micro and electro mechanical systems (MEMS, NEMS) technologies in which nanometre and sub-nanometre resolution is required.

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In the case of displacement measurement, where the resolution is usually given as a fraction of the LD wavelength λ, some efforts have been implemented to reduce this limitation. One of the first efforts is described by Bes[10] with the phase unwrapping method (PUM), which claims that is theoretically possible to reach a resolution of $\lambda/16$. Norgia[11] proposes a method using an electro-optical modulator with results in the order of $\lambda/20$. Wei et al.[12] propose the application of PUM with complex processing methods, such as the use of neural networks, where resolutions as good as $\lambda/25$, in conditions of weak feedback, and of $\lambda/20$, with moderate feedback, were demonstrated. Even when all the mentioned techniques show an increase on the OFI resolution, none of them has proved capabilities of measuring real displacements lower than $\lambda/2$ since all of them rely on acquiring at least one fringe. Trying to overcome this limitation, we propose the application of a differential technique, with which it is theoretically possible to measure nanometre displacements smaller than $\lambda/2$.

The main objective of this paper is to discuss the theoretical basis[13], limitations and amplitude resolution for the displacement measuring technique that further on will be referred as differential optical feedback interferometry (DOFI). In Sec.2 we present the most representative theoretical aspects of the DOFI. Section 3 is devoted to simulation result. On Sec.4 we will discuss some of the practical issues and constraints which can be expected in a laboratory test condition. Finally, the last section is devoted to some of the work that is currently under development and some propositions of future work for this sensor.

2. MEASUREMENT PRINCIPLE

Before discussing the measurement principle of the DOFI, it is important to take into account the effects of the amount of feedback on the OOP since, as it will be shown in sec.3, it will introduce limitations within the detection algorithm proposed in this article.

Depending on the amount of backscattering, the waveform of the OOP will vary as it is shown in Ref. 14. Most typically displacement measurements are processed by applying the so-called fringe counting algorithm to the differentiated waveform of the OOP, from which it is possible to extract the amplitude and the sense of a micrometric displacement. It should be noted that in the fringe counting algorithm each transition corresponds approximately to a $\lambda/2$ displacement.[15] Even though the most common displacement analysis is done with this algorithm, it does not work well in extremely low feedback conditions($C \leq 0.1$), where the OOP behaves as a pure sinusoidal signal. Within the weak and moderate regime ($C \sim 0.3-5$), the modulated waveform changes its appearance and acquires a sawtooth-like form. The transitions in the sawtooth becomes sharper when C reaches a value of 1. For all values of $C \geq 1$ a Hopf-bifurcation process governs the waveform of the OOP whenever the $\lambda/2$ displacement happens, thus, the transitions of the sawtooth are totally vertical. However, this process also induces hysteresis on the waveform. Also, it should be noted that some fringes may be lost starting from the moderate regime as it is shown by Zabit[16]. Finally, with larger values of C a chaotic state governs the feedback signal, thus making it impossible to recover any information.

The most basic setup for DOFI is shown in Fig.1 as it was previously defined in Ref.13. The measuring laser (L_m) is placed on a motorized platform and is aimed to the target. A second laser(L_r) is placed at the same distance from the target to L_m and it is pointed to the platform. When a motion of few micrometres is induced in the platform, both of the sensors capture the OOP related to the motion of the reference. In the case of L_m, a phase change is also induced by the displacement on the target, which creates a difference between both signals. Once the signals are acquired they can be compared and the displacement of the target estimated.

Regarding the lasers, for the analysis shown further on, we will consider that they operate in a moderate feedback level $C \sim 1$. It is desirable that the feedback factor remains within this level to correct the fringe detection and avoid the fringe loss. Temperature effects will be disregarded for the first analysis to get a better notion of the measurement process.

2.1 Frequency analysis

To start this analysis lets consider only the measurement done by L_r. Since the reference motion can be controlled, we can assume two possible type of motions: a sinusoidal motion and a triangular motion. In both cases we can know the number of fringes for a half period which is given by:

$$N = min\left(\frac{2D}{\lambda}\right), \tag{1}$$

where D corresponds to the peak to peak amplitude displacement of the actuator. Since we control the motion of the reference, we can also stablish a known period T for the displacement. Thus, it is clear that if we consider each fringe as a sampling point, the sampling frequency f_s can be estimated:

$$f_s = \frac{2N}{T} = \frac{4f_{ref}D}{\lambda}, \tag{2}$$

where f_{ref} is the frequency of the reference motion.

Even though the same number of fringes can be expected from a triangular and a sinusoidal motion, in the latter one only certain sections of the displacement keep enough linearity as to consider the sampling homogeneous, thus making it mandatory to take out of the model all the measurements around the minimum and maximum position. Hence, for continuous sampling purposes a ramp like reference displacement is more suitable.

From Eq.(2), it is also clear that the sample frequency can be approximated in terms of the reference speed. This calculation can also give us a good idea of the bandwidth of the sensor as well as the cut frequencies that may be applied if any kind of filtering is needed.

2.2 Amplitude resolution

Once the sampling frequency is sorted out, it is necessary to analyse the amplitude resolution that can be reached by the technique. For this study, we will suppose that both lasers have the same wavelength. If, in a first instance, the OOP of both lasers is studied for a fixed target, then the time at which each transition happens should be the same for both lasers. Moreover, if we consider a ramp-like displacement, as proposed earlier, then the time at which two fringes of the same laser happen should be constant. This difference in time can be calculated as:

$$\Delta t_r = t_{r_z} - t_{r_{z-1}}, \tag{3}$$

where t_{r_z} is used to describe the sequential instant z at which a transition of the reference signal happens. The same equation applies for the measuring signal, and for a static target, the direct comparison between Δt_r and Δt_m should be null.

Now, lets consider that the target has some random motion defined as d with a velocity v within the bandwidth previously exposed. Then, as it is shown on Fig.2, the rate of appearance on the transitions for the measurement signal is modified, allowing us to equate the differential velocity for each transition or "sampling point" as:

$$\Delta v = v_m - v_r = \frac{\lambda}{2}\left(\frac{\Delta t}{\Delta t_r \Delta t_m}\right), \tag{4}$$

where $\Delta t = \Delta t_m - \Delta t_r$ is the time difference induced by the target displacement.

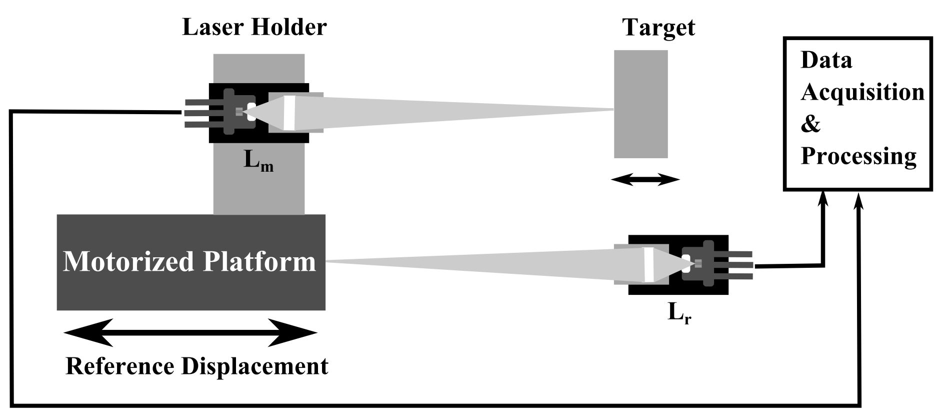

Figure 1. Differential optical feedback scheme. The motorized platform is considered as a linear motion device.

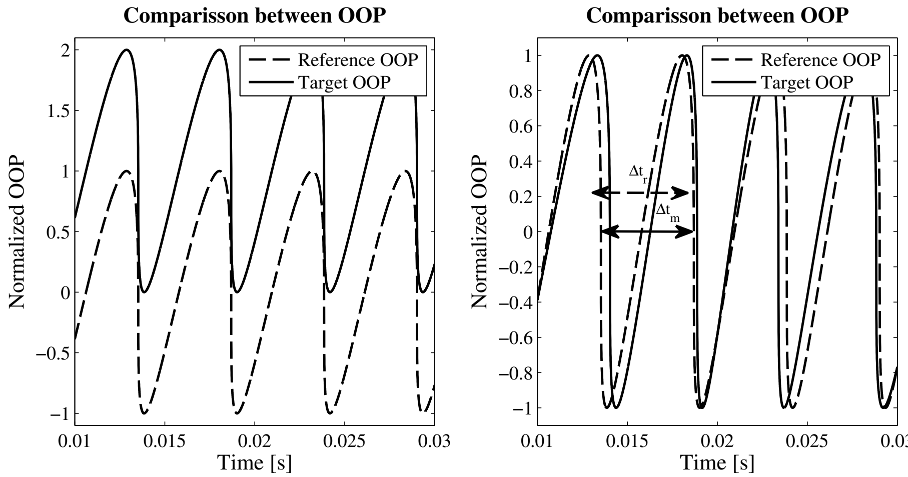

Figure 2. Comparison between signals with a fixed target and a moving target. On the left the fixed target case shows that the transitions occur at the same rate. The vertical axis of the measuring signal is displaced for illustrative purposes. On the right, the motion of the target induces difference on the phase which affects the OOP. For the simulation a 785 nm wavelength and $C \sim 1$ was applied.

Using a simple numerical integration algorithm, it is possible to estimate the displacement for every sampling point as:

$$\Delta d \simeq \frac{\lambda}{2} \frac{\Delta t}{\Delta t_r}. \tag{5}$$

Hence, the resolution of the displacement is theoretically determined by the sampling time of the acquisition device and the time of a transitions due to the reference displacement. As an example, lets suppose we are able to generate a $50\mu m/s$ constant speed displacement for $0.5s$, also we suppose that we have a common acquisition capable of acquiring $250Ksamples/s$, then the theoretical resolution of the sensor is in the order of 1Å.

3. SIMULATION RESULTS

A series of simulations were proposed to observe possible issues with the DOFI measurements. All the simulations were analysed using an algorithm based on the flow chart presented in Fig.3, which is intended for the measurement of real signals. The first set of simulations were performed on the basis of signals without noise to prove if the theoretical results correspond to those proposed in Sec.2. The second set of simulations was performed with a simulated white noise as to have a SNR of 10dB. Three main cases will be analysed for both sets: the difference of C between both lasers, different initial phase and α factor effects. All the simulations were done with L_r and L_m with a 785 nm wavelength.

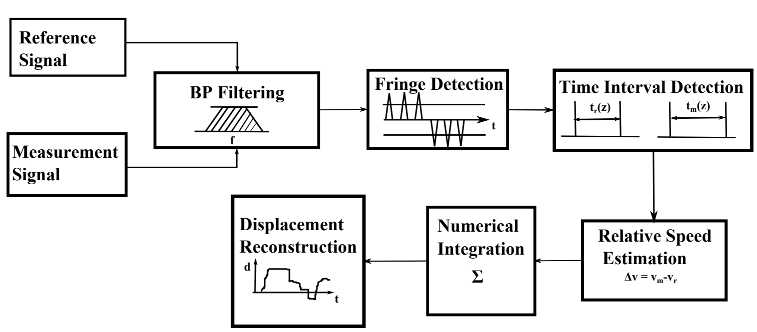

Figure 3. Processing flow chart for DOFI acquisition.

Table 1. Summary of errors obtained after applying the DOFI algorithm to a simulated sinusoidal displacement with a 10nm amplitude.

Test	Mean Error [nm]	Std. Error [nm]
Theoretical Reference	0.1244	0.3034
Theoretical Measurement with Noise	-0.6366	0.6335
$C \sim 0.4$ both lasers	0.3753	0.2111
$C \sim 0.4$ both lasers with noise	-0.2315	0.5471
$C \sim 0.4$ on L_r and $C \sim 0.9$ on L_m	0.6366	0.7447
$C \sim 0.4$ on L_r and $C \sim 0.9$ on L_m with noise	-0.6366	0.6472
α factor	-0.6366	0.7285
α factor with noise	-0.6366	0.7483

As reference displacement a $10nm$ peak to peak amplitude sinusoidal displacement was applied with a frequency of $5Hz$ placed at a $10cm$ distance. For all the cases the number of samples and the relative speed are set as the example presented on Sec.2. A summary of the results is presented on Tab.1.

In general, all the results show discrepancies with the expected resolution. However, the differences are still within the $1nm$, which should be enough to describe any nanometric displacement. Taking into account the standard deviation error of the theoretical simulation, it is possible to say that the resolution of the technique can be estimated as $\lambda/2600$. In general, the higher discrepancies come from noise effects, which would reduce the practical accuracy to values of $\lambda/1000$. This practical value is still 500 times better than the typical OFI and around 40 times better than the PUM technique discussed on Sec.1.

In the theoretical case, the mean difference of the amplitude falls within the expected resolution of 1Å. Nevertheless, if we check the standard deviation we see that the value goes up to 3Å. This difference might be explained because of small differences on the time estimation and numerical errors in the calculation. It should be taken into account that, even in the cases of a 10dB noise, the signal reconstruction has a good agreement with a displacement as the one depicted on Fig.4. A test with a higher number of samples was also performed and a better accuracy for the standard deviation was obtained with a rounded value of 1Å.

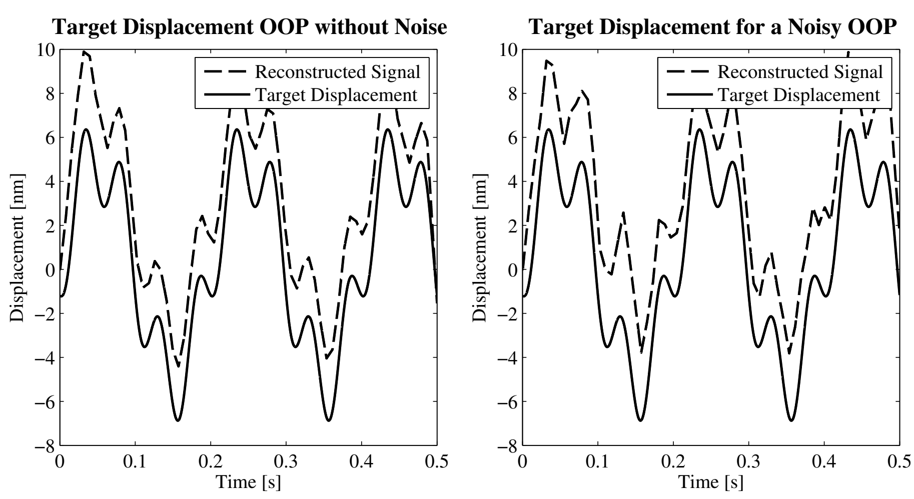

Figure 4. Comparison between reconstructed displacement for a case of a noiseless signal (left), and for the case of a 10dB amplitude noise (right). The offset of the signal is shown only for illustrative purposes.

The rest of the analysed cases, in general, show that noisy conditions will worsen the signal processing, thus limiting the attainable accuracy. Simulations demonstrated that when C value for both lasers are kept around the same order, the DOFI measurement can still be performed with the same order of resolution. Nevertheless, if the feedback factor has a big difference between the two lasers, the slope of the transition may introduce inaccuracies on the fringe time detection. The case of the $\alpha factor$ differences, show similarities with the behaviour of different C factors. From this result, we conclude that the main effect of α is shown by the C coefficient rather than in a direct phase change.

To prove the effect of different starting phase, the initial distance of the target was modified on the simulation up to $9cm$. For this case the standard error also rested bellow $1nm$, still, some time lag effects appeared due to the time at which the fringe happens. For measurement purposes the waveform is the same as in the reference.

4. DISCUSSION

Several factors that could affect the development of a prototype and practical applications have already been discussed on previous sections. Three aspects, however, have not been discussed nor included on the simulation section the effects of the wavelength, thermal effects and calibration requirements.

The wavelength is one of the key aspects to characterize DOFI since any deviation may introduce errors on the velocity estimation. As it was explained for the examples shown on Sec.3, the wavelength was considered equal. In laboratory test conditions, however, this consideration is hardly achievable. Thus, for practical purposes it is necessary to take into account the individual wavelength of each laser. A simulation using Eq.(5) showed that the formula only holds for conditions in which the wavelength difference $\Delta\lambda$ is in the order of few Å. For lager difference values the accuracy of the technique is degraded.

To solve this problem the mathematical model of the wavelength deviation was extended, thus the displacement is estimated as:

$$\Delta d = \pm \frac{\lambda_r}{2} \frac{\Delta t}{\Delta t_r \pm \Delta\lambda \mp \Delta t \Delta t_r}, \tag{6}$$

where $\Delta\lambda = \lambda_m - \lambda_r$ is the difference between the wavelength of the measurement laser λ_m and the wavelength of the reference laser λ_r.

In the model we consider that it is possible to have a reference of the laser wavelengths up to at least the order of 1Å. The model shows that the resolution is approximately kept in the same predicted value as long as the wavelengths are taken into account for the estimation. This point was proven also by simulations obtaining equal results to the ones stated in Sec.3. Other problems that might be introduced by wavelength effects might be the disappearance of fringes, nevertheless, we have only observed this phenomena in simulation for wavelengths with differences higher than $10nm$, which is usually the maximum deviation from the central wavelength in most of the single mode LDs that we have consulted.

As stated before, another possible issue is that of thermal effects. Usually this effects tend to appear at lower frequencies than the OFI signal, thus they should not be a worry for quick measurements. Nevertheless, for long time measurements some deviation of the wavelength may occur, thus limiting the attainable accuracy. To limit the wavelength change due to temperature several strategies can be applied in the form of a temperature control. Even if the temperature control is an appealing solution, the use of low power VCSEL technology is possibly a far more interesting solution. The use of this technology can prove to be a key stone for the production of a DOFI industrial prototype, since usually the spot produced by these LDs is circular and the variations of wavelength due to temperature is quite small when compared with their fabry-perot counterparts.

It is also clear that the DOFI sensor requires a good calibration of the C factor, as well as to take into account the possible variation sources as the ones discussed in this section. To automatize the C factor into an optimum value, it is possible to use the strategy similar to the one described by Atashkhooei[17,18]. This could be done as a previous calibration for any measurement in an industrial process. Linearity of the reference displacement should also be controlled, for this purpose piezoelectric actuators may prove to be the best solution.

5. CONCLUSION

On this article we have presented a new method to measure nanometre scale displacements with sub-nanometre resolution. In the simulations, the DOFI has shown some of its capabilities as well as some limitations that should be addressed to build a robust sensor with industrial capabilities.

Current work is focused on implementing and testing the accuracy of the sensor in a real measurement environment. Some of the tests results acquired so far show good agreement with the theory exposed on this paper, however, further trials to satisfy repeatability within different work environments should be implemented previous to an industrial prototype.

Further work will focus on obtaining experimental results for different testing conditions as well as improving the signal processing algorithm to produce on-line measurements.

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