A Nanometric Displacement Measurement System Using Differential Optical Feedback Interferometry

Francisco J. Azcona, *Student Member, IEEE*, Reza Atashkhooei, Santiago Royo,
Jorge Méndez Astudillo, Ajit Jha

***Abstract*—We propose differential optical feedback interferometry, a technique able to measure nanometer-size amplitude displacements by comparing the optical power of two lasers subject to optical feedback. In this paper, the principles of the technique are explained in detail, and its limits are explored by simulation. Theoretical results are presented which show that the technique can measure nanometer scale displacements with accuracy within the angstrom scale. An experimental setup for validation has been built, and a series of experimental tests were performed using a capacitive sensor as a reference. Results show good agreement between theory and experiment, with a reasonable reduction in performance due to mechanical coupling and signal noise. The proposed technique, thus, provides measurements of very high resolution using an extremely simple and robust experimental setup.**

***Index Terms*—Laser sensors, nanodisplacement sensing, optical feedback interferometry, optical metrology.**

I. INTRODUCTION

THE growing interest on applied nanotechnology processes in recent years has increased the demand for high resolution systems capable of measuring well within the nanometric and sub-nanometric scales. In the case of nanometric measurements, classical interferometry techniques using the phase-shifting method have proven to be a reliable method with accuracies far below 1Å [1]. However, these techniques are only suitable under very strict laboratory environment test conditions, and the systems involved are expensive and require a large number of optical components, as well as careful alignment and calibration.

A more cost-effective and compact solution for the measurement of displacement can be obtained based on optical feedback interferometry (OFI), also known as self-mixing interferometry (SMI). In OFI, light backscattered from a moving target re-enters the laser cavity producing a modulation of the laser optical output power (OOP) [2]. The intensity beats are directly related to changes in the optical path length along the round-trip of the laser to the target. One of the first mathematical models for OFI is described in the classic paper of Lang and Kobayashi [3], where the modulation is explained in terms of the electric field. Later models, like the one presented in [4], use a two cavity Fabry-Perot resonator where the effects of the coupling factor (C) and the linewidth enhancement factor (α) in the OOP of the laser diode (LD) are considered in detail. Two main schemes for the measurement of OFI have been proposed in order to measure the changes in OOP [5]. Usual setups rely on the use of a built-in monitor photodiode (PD), since this type of configuration allows the measurement of changes in the amplitude of OOP in the order of $-90dB$, with reasonable signal to noise ratios (SNR). Alternatively, changes in voltage at the diode junction may be monitored but are limited to detect amplitude differences in the order of $-50dB$.

The main drawback of OFI when compared to traditional interferometry techniques is its resolution, which typically is given as a fraction of the wavelength (λ). The most common displacement detection algorithm for OFI is the fringe counting method, where each fringe is equivalent to a $\lambda/2$ displacement. Some research groups have tried to enhance this resolution using different methods [6]–[8], and resolutions as good as $\lambda/25$ have been reported. Nevertheless, in all of the cases the minimum displacement which could be measured was larger than $\lambda/2$ since at least one fringe is needed to implement the methods. In order to get a better resolution, and to characterize displacements of amplitudes smaller than ($\lambda/2$), we propose the use of a spatial differential technique based on optical feedback interferometry.

In the following section the differential optical feedback interferometry technique (DOFI) is introduced and the basic equations that relate sampling and resolution are detailed. To prove the theoretical resolution value some simulations based on typical experimental values are presented. In section 3, an experimental DOFI setup is used to measure a 40nm amplitude sinusoidal displacement of a piezoelectric stage at different frequencies. The measurement results are compared with those of a built-in internal capacitive sensor. Finally, some conclusions and further work under development will be presented.

II. PRINCIPLES OF DOFI

DOFI is a spatial differential technique in which the delay of a fringe in the measuring laser is used to detect nanometric amplitude displacements of the target. A typical setup for this technique is shown in Fig.1 where the measurement laser (L_m) is mechanically attached to a linear displacement stage (typically, a piezoelectric actuator), and the reference laser

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All the authors are part of the Centre for Sensors, Instruments and Systems Development, Technical University of Catalonia (CD6-UPC), Rambla St. Nebridi 10, 08222, Terrassa, Spain (e-mail: francisco.javier.azcona@cd6.upc.edu; reza.atashkhooei@cd6.upc.edu; santiago.royo@upc.edu; jorge.mendez@cd6.upc.edu; ajit.jha@cd6.upc.edu).

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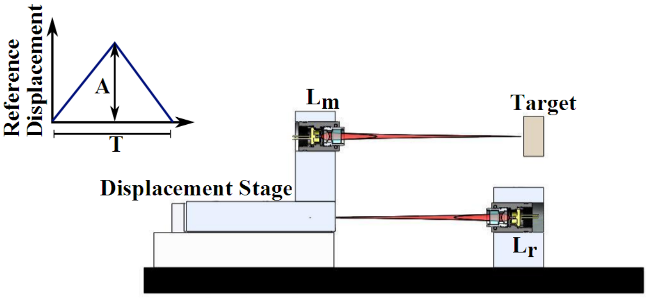

Fig. 1. Standard DOFI setup.

(L_r) is placed facing the stage. When the target is static, both lasers capture only the OFI signals generated by the ramp-like motion of the displacement stage. Therefore, for any motion with amplitude larger than $\lambda/2$ the same number of fringes will appear for each laser OOP. If the target moves with a relative amplitude $d \ll \lambda/2$ then the OOP measured by L_m will experiment a difference on the time at which the fringes happen with respect to the OOP of L_r, thus making it possible to extract the information on the displacement of the target.

A. *DOFI resolution in sampling*

As explained in [9], the sampling frequency of DOFI is not determined by the acquisition device used to detect the OOPs. To have a better understanding of the sampling frequency rate, the OOP waveform produced by a ramp-like or a triangular displacement of micrometric peak to peak amplitude A and an OFI interferometer working with $C \sim 1$ has to be considered. In such case, it can be expected that the transitions of the OOP happen at a constant rate that depends on the constant velocity v of the displacement. Thus, it is possible to write the sampling frequency f_s as:

$$f_s = \frac{v}{\lambda_r} \approx \frac{4Af_r}{\lambda_r} \tag{1}$$

where f_r is the frequency of the reference triangular displacement and λ_r the wavelength of the laser used as reference. Other types of reference displacement can be used (sinusoidal, for instance), although in such cases the sampling is not uniform, making it impossible to quantify the sampling frequency. In the particular case of a sinusoidal reference displacement, it is still possible to get an idea of the sampling frequency, which is only miss-evaluated in the regions near the maxima and minima of the reference displacement.

Therefore, the bandwidth of the DOFI technique is directly proportional to the speed of the reference, and it is only limited by the mechanical specifications of the device used as reference.

B. *DOFI resolution in amplitude*

In the case of the resolution in amplitude, two factors directly affect the quality of the measurement: wavelength and data acquisition rate. To ease the analysis, it is possible to assume that both lasers have the same wavelength and that the target is initially static. In such a case, the OOPs of L_m and L_r are equal. If the target starts moving, then some of the transitions on the OOP of L_m appear faster than expected or are delayed from the reference, as shown on Fig. 2.

It should be noted that the transitions in both OOPs only occur when a $\lambda/2$ difference on the optical path happens. Thus, it is possible to recover the discrete displacement information as a function of the delay in time $\Delta t = \Delta t_m - \Delta t_r$. Moreover, due to the differential nature between displacement and velocity, it is possible to calculate the discrete displacement with two lasers that have a small difference of wavelength as:

$$\Delta d = \frac{\lambda_r}{2} \frac{\Delta t}{\Delta t_r \left(1 - \dfrac{\Delta\lambda}{\lambda_r}\right) - \Delta t} \tag{2}$$

where $\Delta\lambda = \lambda_m - \lambda_r$ behaves as a correction factor introduced to compensate for the difference in wavelength between L_r and L_m, and Δt_r is the time elapsed between two consecutive transitions on the reference OOP.

It is possible to estimate the attainable resolution by taking into account a typical digital to analog converting rate and the reference displacement. Therefore, a slower displacement results in a higher resolution in amplitude. Thus, for a correct reconstruction it is necessary to ponder the velocity of the reference displacement since it affects sampling as well as amplitude resolution.

III. METHODS AND RESULTS

To prove the feasibility of the method a series of simulations and laboratory measurements were proposed. In all cases the reference displacement is selected as a ramp-like positive motion with amplitude of $38\mu m$ and a reference speed of $76\mu m/s$ during $0.5s$.

A. *Simulation results*

A series of simulations were proposed in order to prove the feasibility of the method, its performance and its limitations. Simulations were performed using typical performance values of electromechanical equipment. A theoretical displacement waveform reconstruction is shown in Fig.3. As it is shown, the reconstructed waveform shows the effect of the sampling frequency discussed on Sec.II-A. If the sampling frequency

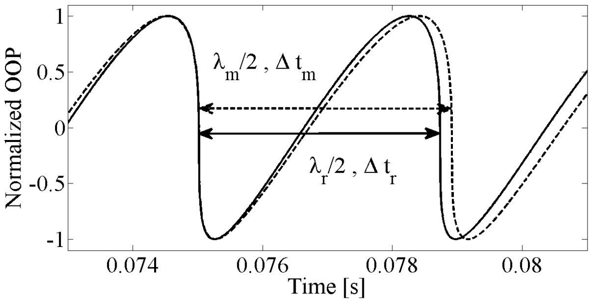

Fig. 2. Time frame comparison of the delay between two transitions in the reference and measurement OOPs. Reference OOP in continuous line and target measurement OOP in dashed line.

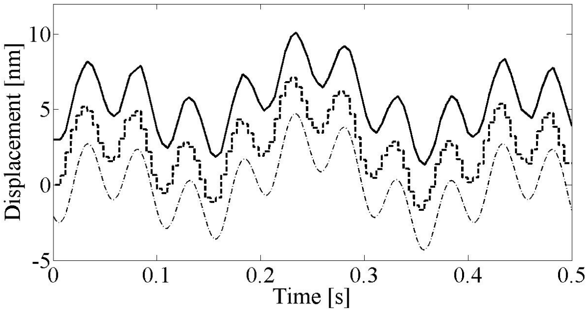

Fig. 3. Comparison between simulated displacements (thin dashed line), rough DOFI reconstruction (dark dashed line) and linearly interpolated DOFI reconstruction (continuous line).

rate chosen is high, it is possible to use linear interpolation algorithms in order to smooth the waveform.

Four factors in the experiment were considered with special care in the simulations: theoretical resolution, the effects of measuring with OOPs in the weak regime ($C \sim 0.3$), the effects of measuring two OOPs with different feedback factors, and the effects of white noise significantly reducing SNR. A summary of the results obtained is found in Tab.I, where it is patent that two main factors can affect the final resolution of the technique: differences in the coupling factor C of both lasers and reduced SNR values. In practical conditions, the coupling factor issue can be controlled by adjusting the lasers to be in the boundaries of weak and moderate regimes, as discussed in [10], [11]; by using this type of control it is also possible to reduce speckle effects as it is described in [12]. The effect of using different wavelengths was also studied, although no significant difference was observed in terms of amplitude detection. The most relevant effect of using different wavelengths resulted in the appearance of a time shift in the reconstructed waveforms.

B. Experimental results

In order to test the attainable accuracy of the DOFI method in laboratory conditions, a prototype was built (Fig.4). The prototype setup is composed by two $HL7851G$ laser diodes with an adjustable focusing lens calibrated to be in the boundary between weak and moderate feedback regimes to avoid any possible fringe loss. The measurement laser was mechanically attached to a PI-LISA piezoelectric displacement stage with nominal amplitude of $38\mu m$. The reference laser was placed facing the reference stage at an approximated distance of $10cm$. A second PI-LISA stage with an internal capacitive sensor and nominal displacement amplitude of $25\mu m$ was chosen as target. The capacitive sensor has a resolution of $2nm$ and is used for validation purposes. The stage was placed approximately at $10cm$ from the measuring laser. All the electrical signals were recovered using a Tektronics DPO2024 oscilloscope with $125kS$ window.

TABLE I
TYPICAL ERRORS OBTAINED IN DOFI UNDER DIFFERENT SIMULATED EXPERIMENTS.

Test	Mean Err. [nm]	Std. Dev. [nm]	Mean Err. [nm]	Std. Dev. [nm]
	Rough Reconstruction		Linear interpolation	
Theoretical	0.4295	0.5533	0.1289	0.1395
Low C	0.7164	0.9132	0.6399	0.8215
Different C	0.7537	0.8994	0.6780	0.7294
$SNR \sim 2$	0.8073	1.0039	0.6752	1.0039
$SNR \sim 0$	1.6917	2.0092	1.6489	1.9043

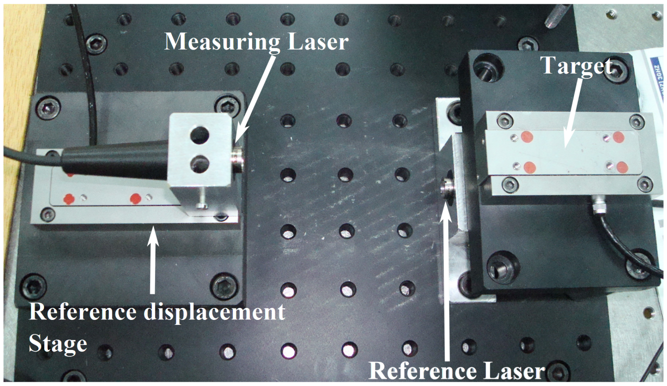

Fig. 4. DOFI experimental prototype setup.

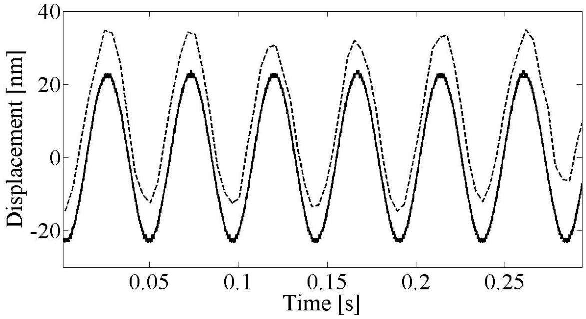

Fig. 5. Typical DOFI reconstructed displacement (dashed line) and capacitive sensor response (continuous line).

The displacement of the target was chosen as a sinusoidal motion with fixed peak to peak amplitude of 40 nm and frequency of 5, 10, 20 and $50Hz$ respectively. The measurements were performed 10 times for each frequency to test the repeatability of the measurement. When compared with the capacitive sensor readings, the reconstructed waveforms show standard deviation in the order of $5nm$. A summary of the average errors and average standard deviation values is shown in Tab.II. The set of results show that experimentally it is possible to reconstruct displacements with a resolution of $\lambda/100$.

IV. CONCLUSION

A prototype system based on OFI capable of measuring nanometric displacements was proven theoretically and experimentally. A difference of one order of magnitude can be observed between theory and experiments. A possible explanation of these differences is the presence of mechanical

TABLE II
AVERAGE MEAN ERROR AND STANDARD DEVIATION VALUES OF INTERPOLATED DOFI .

bfseries Frequency [Hz]	**Avg. Err. [nm]**	**Std. Dev. [nm]**
5	1.05	6.91
10	0.75	4.24
20	0.45	2.71
50	0.46	4.88

perturbations, which can be seen particularly at low frequencies because of the mechanical coupling between elements in the experiment. In the $50Hz$ case, the difference comes from sampling rate limitations which can be corrected by changing the speed of the reference displacement. With our current prototype it is possible to measure displacements up to $20kHz$ with resolutions within $4nm$.

Further work needs to be performed before building a suitable industrial prototype. The reduction of mechanical coupling in the measuring system is one of the points to address since it can prove to reduce the errors in the measurement, particularly on the low frequency range. Signal processing and data acquisition systems also need to be enhanced in order to produce on line measurements as well as to enable the measurement of longer events. In order to increase bandwidth, a modification of the technique is already being studied and theoretical results show that measurements within the MHz range are feasible.

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REFERENCES

[1] A. Bergamin, *et al.*, “A displacement and angle interferometer with subatomic resolution, *Rev. Sci. Instrum.*, vol.64, no.11, pp. 3076 3081, 1993.

[2] W. M. Wang, *et al.*,“Self-mixing interference in a diode laser: experimental observations and theoretical analysis,” *Appl. Opt.*, vol.32, no.9, pp. 1551 - 1558, 1993.

[3] R. Lang and K. Kobayashi, “External optical feedback effects on semiconductor injection laser properties,” *IEEE J. Quantum Electron.*, vol.16, no.3, pp. 347 - 355, 1980.

[4] W. M. Wang, *et al.*, “Self-mixing interference inside a single-mode diode laser for optical sensing applications,” *J. Lightw. Technol.*, vol.12, no.9, pp. 1577 - 1587, 1994.

[5] S. Donati, “Responsivity and noise of self-mixing photodetection schemes,” *IEEE J. Quantum Electron*, vol.47, no. 11, pp. 1428 - 1433, 2011.

[6] C. Bes, *et al.*, “Displacement measurements using a self-mixing laser diode under moderate feedback,” *IEEE. Trans. Instrum. Meas.*, vol.55, no. , pp. 1101 - 1105, 2006.

[7] N. Servagent, *et al.*, “Design of a phase-shifting optical feedback interferometer using an electrooptic modulator,” *IEEE J. Sel. Topics Quantum Electron.*, vol. 6, no. , pp. 798 - 802, 2000.

[8] L. Wei, *et al.*, “Phase unwrapping of self-mixing signals observed in optical feedback interferometry for displacement measurement,” in *2006 International Symposium on Intelligent Signal Processing and Communication Systems*, Tottori, Japan, 2006, pp. 780-783.

[9] S. Royo, *et al.*, “A method of measuring a displacement related parameter using a self-mixing laser measuring system and a laser self-mixing measuring system,” PCT Patent 2012/049561, April 19, 2012.

[10] U. Zabit, *et al.*, “Adaptive self-mixing vibrometer based on a liquid lens,” *Opt. Lett.*, vol.35, no.8, pp. 1278 - 1280, 2010.

[11] R. Atashkhooei, *et al.*, “Analysis and control of speckle effects in self-mixing interferometry,” in *IEEE Sensors 2011*, Limerick, 2011, pp. 1390 - 1393.

[12] R. Atashkhooei, *et al.*, “Dealing with speckle effects in self-mixing interferometry measurements,” *IEEE Sensors J.*, vol.13, no.5, pp. 1641 - 1647, 2013.