Coupling-mediated ghost resonance in mutually injected lasers

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We experimentally and numerically study the phenomenon of ghost resonance in coupled nonlinear systems. Two mutually injected semiconductor lasers are externally perturbed in their pump currents by two respective periodic signals of different frequencies f_1 and f_2. For small amplitudes of the external modulations, the two laser intensities display synchronized optical pulses, in the form of dropout events occurring at irregular times. By adjusting the amplitude and frequencies of the driving signals, the system exhibits a ghost resonance in the dropout appearance at a frequency f_r not present in the distributed inputs. © *2005 American Institute of Physics.*
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A classical phenomenon associated with the detection of complex signals is the "missing fundamental illusion," in which a listener subject to an acoustic signal composed of two tones hears a lower tone not present in the input. A simple mechanism explaining this effect at the single-neuron level has been recently proposed, involving a linear superposition of the input tones and a nonlinear detection (aided by noise) of the resulting total signal by the neuron. This mechanism, which has been termed *ghost* stochastic resonance, produces good agreement with existing physiological data, and has been confirmed as well in *ad hoc* experiments in excitable electronic circuits and lasers. Here we propose a nontrivial extension of this phenomenon, in which the two input signals act upon *different* nonlinear systems, interfering nontrivially via the coupling that exists between the systems, and also leading to the detection of a ghost frequency. This situation is observed experimentally in two mutually coupled semiconductor lasers, and reproduced numerically with a model of the coupled laser dynamics. We believe that this effect could be of relevance, and occur generically, in other types of coupled excitable systems, such as neuronal networks.

I. INTRODUCTION

Nonlinear systems frequently exhibit nontrivial resonances in response to external perturbations. Excitable systems, in particular, have been profusely studied in this respect, due to their sensitivity to external driving[1] and to the existence of excitable dynamics in many different contexts, such as in biological systems,[2] chemical reactions,[3] electronic circuits[4] and optical devices.[5] Sensory neurons, for instance, substantially modify their firing patterns when periodically forced by a single sinusoidal modulation.[6]

Recently the phenomenon of *ghost resonance* (GR) has been reported, in which an excitable system subject to two different periodic signals exhibits a resonance at a frequency not present in the input driving.[7] This phenomenon, invoked to explain the missing fundamental illusion arising in the perception of complex sounds,[8] has been shown to be produced by the interplay between noise and periodic forcing in a nondynamical threshold device[7] and in an electronic circuit,[9] but it can also be caused by the application of periodic perturbations to a chaotic system. The latter effect has been experimentally observed in a semiconductor laser with optical feedback operating in the low-frequency fluctuation regime.[10] Under this particular regime of operation, the laser has been shown to have excitable properties,[11] responding with intensity dropouts to pump perturbations only when the perturbation amplitude surpasses a certain threshold. Nevertheless, and due to the complexity of this particular system,[12] such excitation threshold is not a well defined constant value, but varies dynamically depending on the particular trajectory of the laser at every instant. When, under this scenario, two different periodic signals are introduced into the system via the laser's pump current, a resonance appears at a frequency not present in the input, in an example of GR.[10] In this case, the interplay between the external modulation and the complex dynamical threshold of the system shows a response similar to the effect of a combined periodic-noisy external signal on a simpler excitable system with a well-defined excitation threshold.[7]

The occurrence of GR in a single dynamical system relies on the joint action of two mechanisms: a linear superposition of the harmonic inputs taking place *outside* the system, and a nonlinear detection process of the resulting total input signal. A nontrivial extension of this phenomenon corresponds to a situation where the inputs act in a distributed way on different nonlinear systems coupled to one another, in such a way that the signal superposition dwells instead *within* the coupled system (and is no longer linear). With the aim of investigating this scenario, in the present paper we study two coupled lasers driven separately by a distinct external perturbation each, and show that the joint system can resonate at a third frequency different from those of the input signals. In other words, the GR in this case is *mediated by the coupling* between the dynamical elements.

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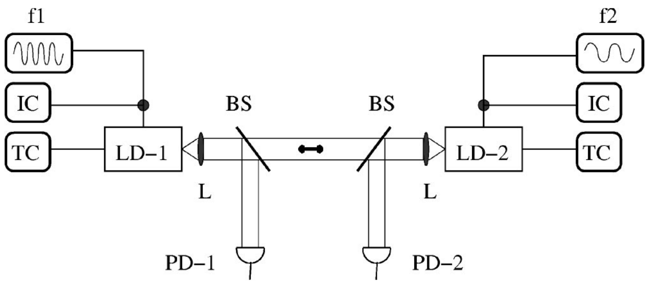

FIG. 1. Schematic setup: LD-1 and LD-2 are the laser diodes; TC and IC are the temperature and current controllers, respectively; L are collimating lenses; BS are beamsplitters; PD-1 and PD-2 are photodetectors.

Specifically, we consider two semiconductor lasers coupled bidirectionally via the mutual injection of their output intensities. This coupled system has been shown to have a pulsated output,[13] similar to that observed in a single semiconductor laser with optical feedback in the low-frequency fluctuation regime. Previous studies have shown that coupling in this system enhances the response of both lasers to a single periodic driving of one of the lasers.[14] We will now show that when both lasers are independently modulated, coupling induces entrainment to a frequency not present at the input of any of the two lasers. We note that coupling has a second role in this system, namely inducing the excitable behavior itself: In the absence of coupling the lasers exhibit a stable output. Therefore, the results presented in what follows are also evidence that coupling can induce genuine excitable behavior (exemplified in this case by the phenomenon of ghost resonance) in interacting dynamical systems.

II. EXPERIMENTAL RESULTS

Our experimental system consists of two AlGaInP index-guided and multi-quantum well semiconductor lasers (Roithner RLT6305MG), LD-1 and LD-2 in what follows, mutually injected as shown in Fig. 1. Both lasers have a nominal wavelength of $\lambda_n = 635$ nm. Temperature and pump current of the lasers are controlled with an accuracy of ±0.01 °C and ±0.1 mA, respectively. For temperatures $T_{\mathrm{LD-1}} = 19.80$ °C and $T_{\mathrm{LD-2}} = 17.98$ °C, their threshold currents (in isolation) are, respectively, $I^{\mathrm{th}}_{\mathrm{LD-1}} = 18.3$ mA and $I^{\mathrm{th}}_{\mathrm{LD-2}} = 18.0$ mA. The operating currents are set to $I_{\mathrm{LD-1}} = 18.9$ mA and $I_{\mathrm{LD-2}} = 19.4$ mA. The relative pump currents are slightly different for both lasers, but this small asymmetry does not have an important influence in the results that follow. To quantify the effect of the opposite laser as a source of external optical feedback, we estimate the threshold reduction of each laser when the opposite laser is turned off, obtaining a reduction of 3.79% for LD-1 and 1.20% for LD-2. When both lasers are turned on, the threshold current is also decreased, but due to the interaction between the fields the threshold reduction in this case is 4.50% for LD-1 and 5.70% for LD-2. Two beamsplitters in the coupling branch allow the detection of the laser outputs by fast photodetectors of 1 GHz bandwidth (Thorlabs DET210), and the received signal is sent to a 5 GS/s acquisition card (Gage 85G). The external periodic signals are introduced by modulating the pumping current of the lasers with two Agilent 33250A signal generators.

Under the conditions described above the lasers are synchronized, which means that when an external perturbation is introduced into the system, both lasers respond in the same way. In the conditions of our experiment the lasers remain stable in the absence of external perturbations,[7] but when external sinusoidal signals are introduced through their pumping currents, intensity dropouts appear. The dropouts are synchronized with a delay time between them corresponding to the coupling time $\tau_c = 3.43$ ns between the lasers (time that the light takes to travel the distance between the lasers). In other words, the dropouts of one laser advance those of the other a time interval τ_c. In the absence of frequency difference between the lasers, they randomly alternate the leading role in the dynamics. To avoid this effect, which makes difficult to estimate the correlation between both output intensities, a slight frequency detuning has been introduced by tuning the operating temperatures, since the laser with higher optical frequency is known to lead the dynamics.[13]

In order to establish the existence of a ghost resonance in this system, we apply to the pump current of LD-1 (LD-2) a periodic modulation of frequency f_1 (f_2). The input frequencies have the general form[7]

$$f_n = (k + n - 1) f_0 + \Delta f, \quad n = 1, 2, \tag{1}$$

where $k > 1$ is an integer and Δf is a frequency detuning. An analysis of the conditions of constructive interference arising from the superposition of the frequencies given in Eq. (1) shows that the expected resonant frequency is (see Ref. 7 for details)

$$f_r = f_0 + \frac{\Delta f}{k + 1/2}. \tag{2}$$

We consider first the simplest case $k=2$ and $\Delta f = 0$. The external sinusoidal frequencies are $f_1 = 10$ MHz and $f_2 = 15$ MHz. Then, from Eqs. (1) and (2) we see that the resonance frequency is $f_r = f_0 = 5$ MHz. Figure 2 shows the output intensity of LD-1 [Figs. 2(a)–2(c)] and the corresponding probability distribution function (PDF) of the time interval between consecutive dropouts [Figs. 2(d)–2(f)] for increasing values of the modulation amplitude (which is the same for both sinusoidal signals). The dynamics of LD-2, not shown, is identical to that of LD-1, since the two lasers are synchronized.

The results of Fig. 2 show that for low values of the modulation amplitude [Figs. 2(a) and 2(d)] the output intensity exhibits dropouts, but they are not distributed regularly; the corresponding PDF of the inter-dropout intervals exhibits several peaks at multiples of the interval $T_r = 200$ ns corresponding to the ghost frequency $f_r = 5$ MHz. For intermediate values of the modulation amplitude [Figs. 2(b) and 2(e)], the system shows a well-defined resonance at the ghost frequency, characterized by a single sharp peak of the PDF at the ghost interval $T_r = 200$ ns. When the modulation amplitude is further increased [Figs. 2(c) and 2(f)], the peak at the ghost resonance period diminishes and the system shows intensity dropouts at the input frequencies f_1 and f_2.

In order to ensure that a resonance occurs at $f_r = 5$ MHz, we statistically analyze the output times series

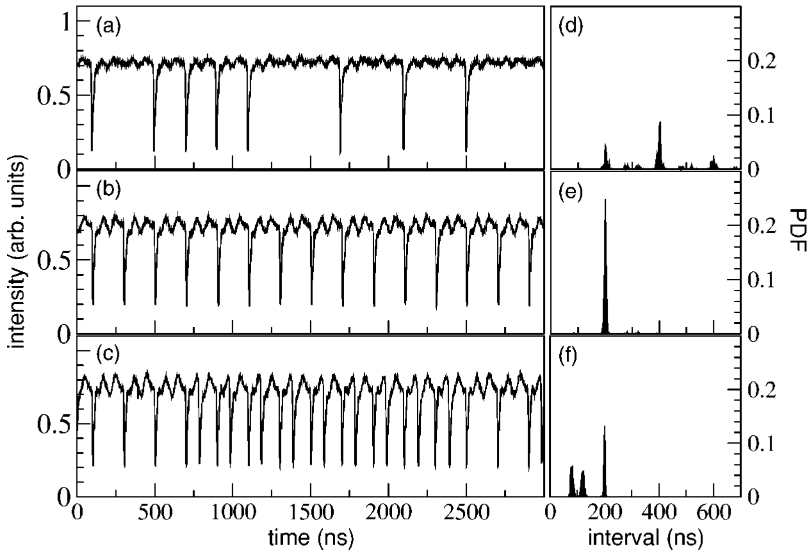

FIG. 2. Experimentally observed output intensity of LD-1 (left column) and the corresponding probability distribution functions (right column) for increasing values of the modulation amplitude: $A_1 = A_2 = 0.191$ mA [(a) and (d)] $A_1 = A_2 = 0.409$ mA [(b) and (e)] and $A_1 = A_2 = 0.575$ mA [(c) and (f)].

(with more than 10^3 dropouts) for different modulation amplitudes, evaluating the average and the relative standard deviation of the inter-dropout intervals at each amplitude. Figure 3 shows how the standard deviation is minimal for average inter-dropout frequency $f_r = 5$ MHz, which indicates that the periodicity is maximal at that output frequency.

The previous results do not correspond to a trivial resonance at the difference between f_1 and f_2. To demonstrate this, we now introduce a frequency detuning Δf in the input frequencies according to Eq. (1). Such detuning renders the two input frequencies incommensurate. The prediction of Eq. (2) indicates that in this case the resonance frequency increases linearly with the detuning, even though the difference between the input frequencies is still f_0. Experimentally, f_0 is kept at 5 MHz and f_1 is increased from $2f_0 = 10$ MHz to $3f_0 = 15$ MHz in steps of 0.5 MHz, while the modulation amplitudes are kept constant at $A_1 = A_2 = 0.409$ mA. Figure 4 shows the resulting PDFs (vertically, in solid lines) for increasing values of f_1, and the theoretical resonance frequencies predicted by Eq. (2), in dashed lines. The experimental PDFs are plotted vertically vs the response frequency f_{res} (i.e., inverse of the inter-dropout interval), and they are lined up horizontally with respect to the frequency f_1 at which they were obtained. We can observe how the maxima of the experimental PDFs, which correspond to the resonance frequency f_r, shift with Δf according to the theoretical prediction[15] of Eq. (2), thus demonstrating the existence of a nontrivial GR.

Figure 4 also shows that the resonant response of the system (measured in terms of the height of the PDF peaks) strongly depends on f_1. This is due to the general fact that the response of semiconductor lasers (in particular in the low-frequency-fluctuation regime) to external current modulation exhibits a nontrivial dependence on the modulation frequency (see Ref. 16 for details).

III. NUMERICAL SIMULATIONS

The experimental results described above can be numerically reproduced by means of the following system of delay-coupled rate equations[17]

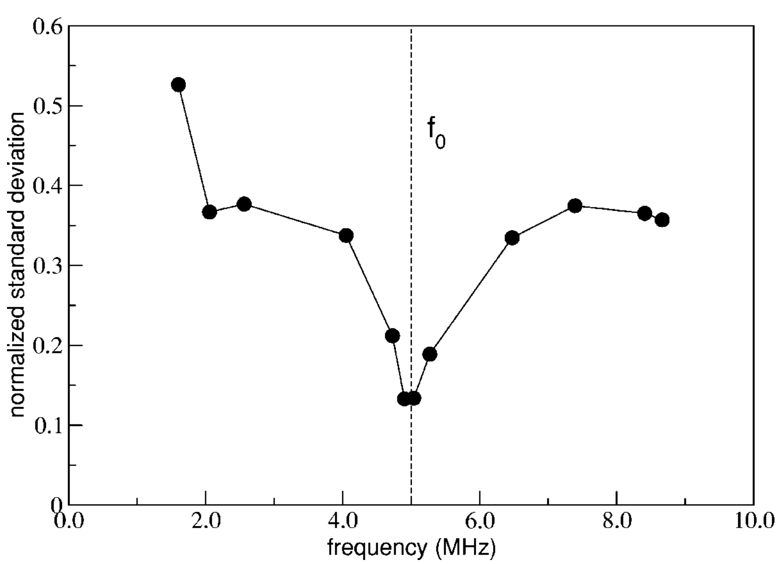

FIG. 3. Standard deviation of the interval between dropouts (normalized to the mean interval) vs the mean inter-dropout frequency.

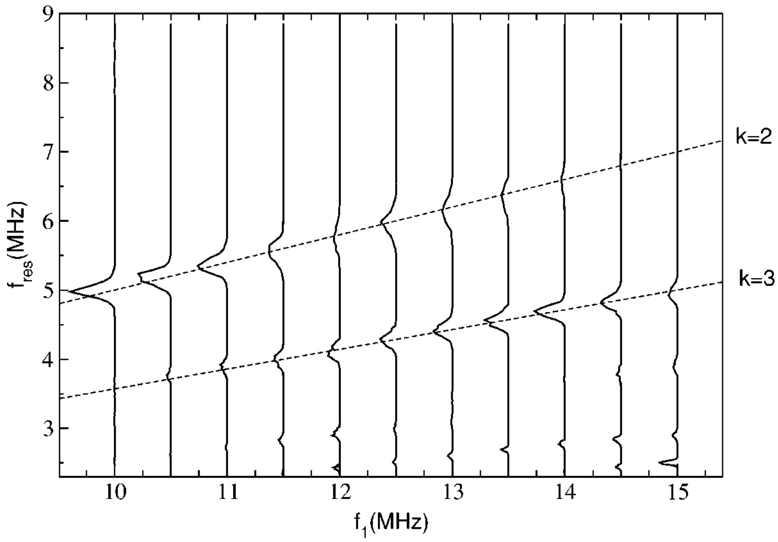

FIG. 4. Experimental PDFs vs the response frequency f_{res} (the inverse of the dropout interval) for increasing values of f_1, with $f_2=f_1+f_0$ and $f_0=5$ MHz. The dashed lines correspond to the theoretical resonance frequencies given by Eq. (2), with $\Delta f=f_1-kf_0$.

$$\frac{dE_{1,2}}{dt}=\frac{(1+i\alpha)}{2}[G_{1,2}-\gamma_{1,2}]E_{1,2}+\kappa_c e^{i\omega_{2,1}\tau_c}E_{2,1}(t-\tau_c)+\sqrt{2\beta N_{1,2}}\xi_{1,2}(t), \quad (3)$$

$$\frac{dN_{1,2}}{dt}=\frac{I_{1,2}}{e}-\gamma_{e1,2}N_{1,2}-G_{1,2}P_{1,2}(t), \quad (4)$$

where $E_{1,2}$ represent the optical fields of LD-1 and LD-2 and $N_{1,2}$ their corresponding carrier number. $\omega_{1,2}$ are the free-running optical frequencies of the two lasers, which for simplicity are considered to be the same. The optical intensity (or number of photons inside the cavity) is given by $P_{1,2}(t)=|E_{1,2}(t)|^2$. In the first term of the right-hand side of Eq. (3) α is the linewidth enhancement factor (assumed equal for both lasers), $\gamma_{1,2}$ is the photon inverse lifetime and $G_{1,2}(t)=g_{1,2}(N_{1,2}-N_{1,2}^0)$ is the nonlinear gain, where $N_{1,2}^0$ denote the carrier number at transparency for each laser and $g_{1,2}$ their differential gain (gain saturation is neglected because the lasers operate close to threshold). The second term of the r.h.s. of Eq. (3) accounts for the bidirectional coupling between the lasers, with κ_c representing the coupling strength and τ_c the coupling time. The last term of this equation corresponds to spontaneous emission noise, where β is the spontaneous emission rate and $\xi_{1,2}(t)$ is a Gaussian white noise of zero mean and unity variance.

In the simulations that follow we have set $\alpha=3.0$, $\beta=0.5\times10^{-9}$ ps^{-1}, and $\kappa_c=3.0$ ns^{-1}. The rest of the laser parameters have been chosen to reproduce the experimental conditions, including the threshold currents $I_{LD-1}^{th}=18.30$ mA and $I_{LD-2}^{th}=18.02$ mA, and the coupling time $\tau_c=3.43$ ns. Their values are $\gamma_1=0.50$ ps^{-1}, $\gamma_2=0.48$ ps^{-1}, $\gamma_{e1}=6.89\times10^{-4}$ ps^{-1}, $\gamma_{e2}=6.72\times10^{-4}$ ps^{-1}, $g_1=1.20\times10^{-9}$ ps^{-1}, $g_2=1.18\times10^{-9}$ ps^{-1}, $N_1^0=1.25\times10^8$, and $N_2^0=1.27\times10^8$. Finally, the pump currents take the form $I_{1,2}=I_{DC1,DC2}[1+A_{1,2}\sin(2\pi f_{1,2}t)]$, where $I_{DC1,DC2}$ are the DC pump currents, $A_{1,2}$ the amplitudes of the modulations and $f_{1,2}$ their corresponding frequencies, chosen again following Eq. (1). The DC levels are chosen to be $I_{DC1}=1.032\times I_{LD-1}^{th}$, $I_{DC2}=1.076\times I_{LD-2}^{th}$.

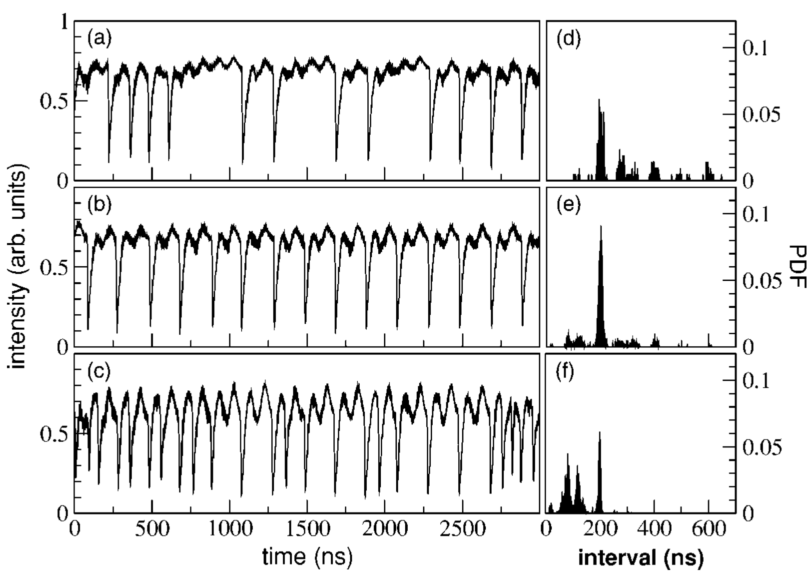

FIG. 5. Numerically computed output intensity of LD-1 (left column) and the corresponding probability distribution functions of the inter-dropout intervals (right column) for increasing values of the modulation amplitude: $A_1=A_2=0.013$ [(a) and (d)], $A_1=A_2=0.020$ [(b) and (e)], and $A_1=A_2=0.045$ [(c) and (f)].

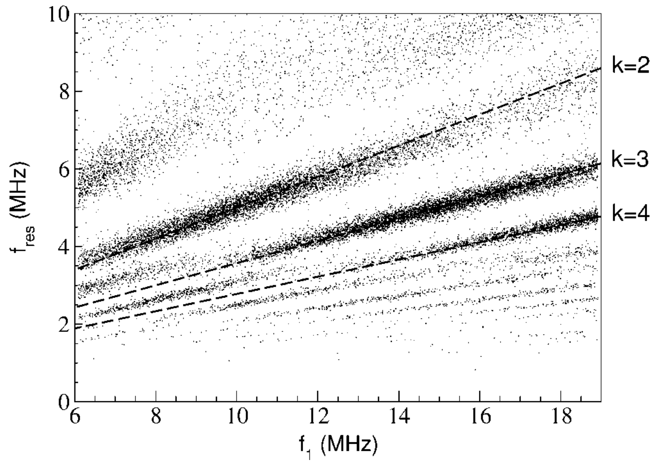

FIG. 6. Numerically determined response frequency between dropouts f_{res} as a function of f_1, with $f_2=f_1+f_0$. Dashed lines correspond to theoretical predictions given by Eq. (2) for $k=2\ldots4$.

In order to reproduce the experimental results presented in Fig. 2 for the case $\Delta f=0$, we fix the input frequencies to $f_1=10$ MHz and $f_2=15$ MHz, increasing the amplitude of both modulations simultaneously. The results can be seen in Fig. 5, where the ghost resonance at the frequency $f_r=5$ MHz is observed for intermediate values of the modulation amplitudes (note the qualitative resemblance with Fig. 2).

Finally we analyze the effect on f_r of a detuning Δf introduced in both modulation frequencies following Eq. (1). To that end, we compute the response frequencies f_{res} of 25 dropouts for increasing values of f_1, where f_1 is incremented in steps of 0.001 MHz (with $f_2=f_1+f_0$, keeping $f_0=5$ MHz). Figure 6 shows the response frequency f_{res} as a function of f_1, and also the comparison with the theoretical predictions of Eq. (2) (dashed lines). We can observe how, as f_1 increases, the maximum response frequencies (resonance frequencies) jump from the theoretical value of f_r to a value of f_r corresponding to a higher k parameter. This is in agreement with the experimental PDFs represented in Fig. 4, where the PDF maxima jump from the $k=2$ to $k=3$ line when f_1 is increased. This dependence of f_r with Δf indicates that the GR is a nontrivial resonance.

IV. CONCLUSIONS

Excitable systems are often embedded in networks where many excitable units are *coupled* to each other and respond collectively to external perturbations. Sensory neurons, for instance, do not usually operate in isolation.[18] The question then arises as to what happens when external signals of different frequencies act on *different* elements within the network. Here we have shown experimentally and numerically that when two mutually coupled semiconductor lasers are perturbed by two independent periodic signals, a resonance arises at a third frequency not present in the input, what is known as a ghost resonance. The resonance is not trivial, since it persists in the case of incommensurate input frequencies, showing a behavior that agrees with theoretical predictions.

In the scheme reported here coupling has the additional role of inducing the excitable behavior itself (the lasers are stable without coupling) but preliminary results, to be published elsewhere, show that the ghost resonance also arises in coupled excitable lasers (namely, when the lasers are subject to optical feedback).

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[15]Note that the frequency range in the horizontal axis of Fig. 4 is such that Δf is positive for the $k=2$ line, whereas it is negative for the $k=3$ one.
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