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Synchronization of delay-coupled semiconductor lasers

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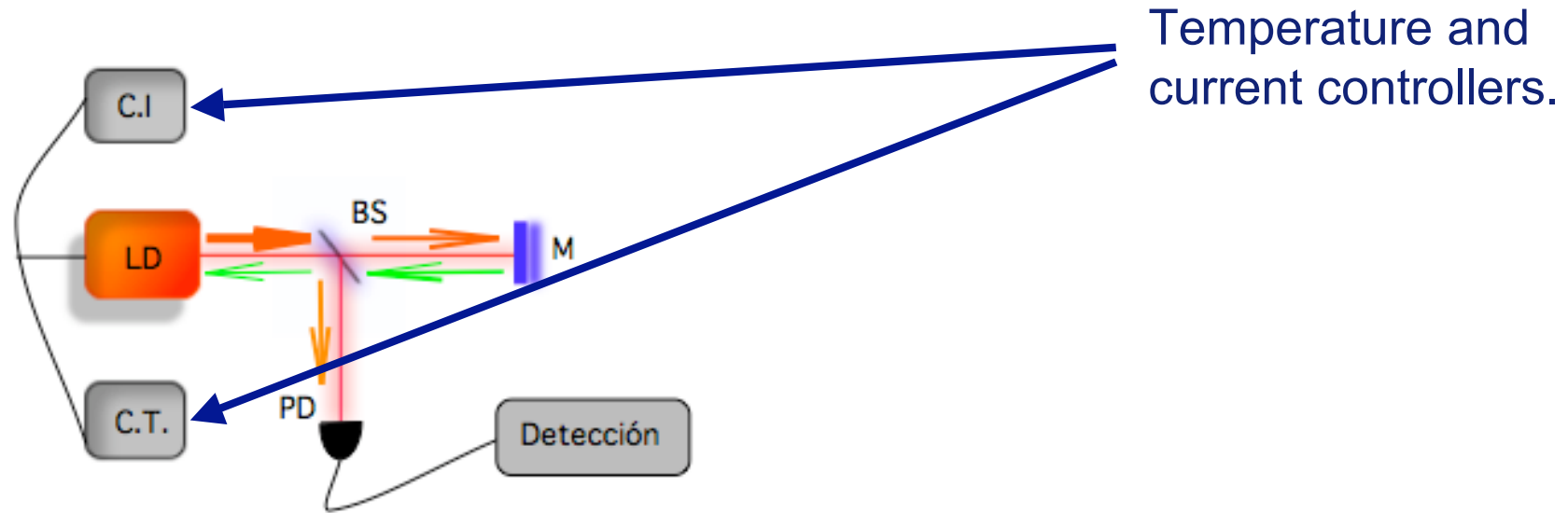
J. García-Ojalvo



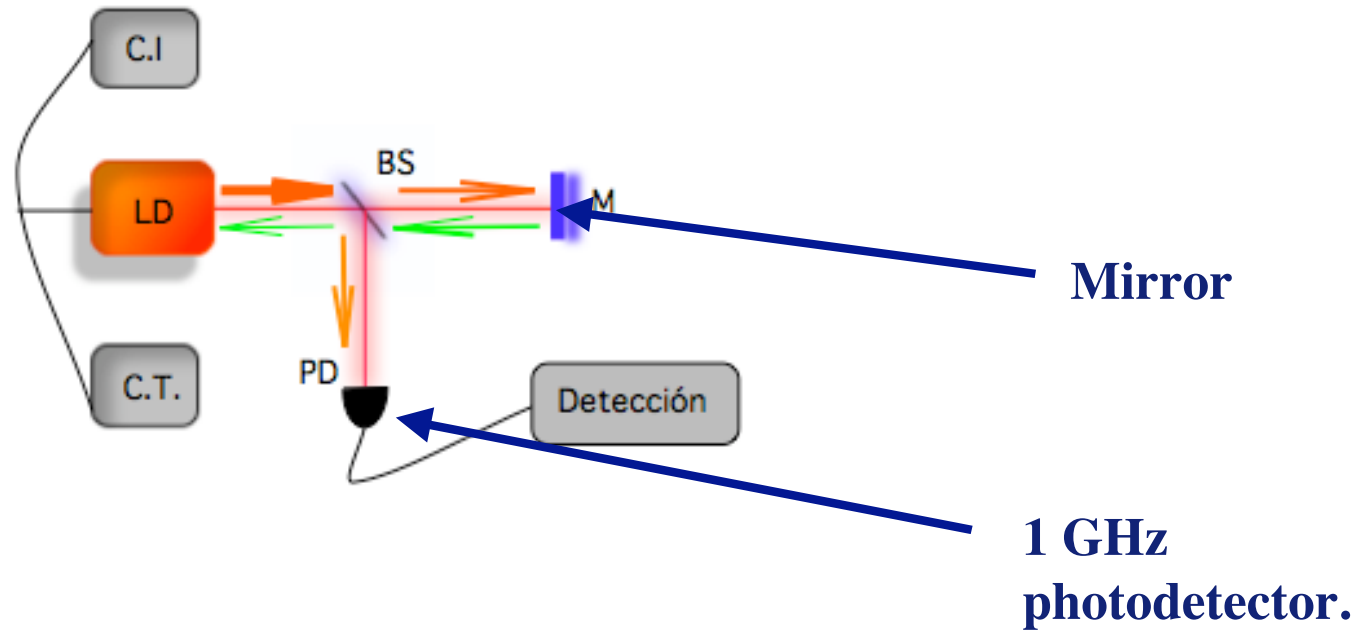
Outline

- Introduction.
 - ✓ Induction to chaos in semiconductor lasers.
 - ✓ One laser: Optical feedback.
 - ✓ Two lasers: Optical injection.
- Control of leader-laggard dynamics in **two coupled** semiconductor lasers (experimental and numerical).
- Route to synchronization with **three coupled** semiconductor lasers (experimental and numerical).
- Conclusions.

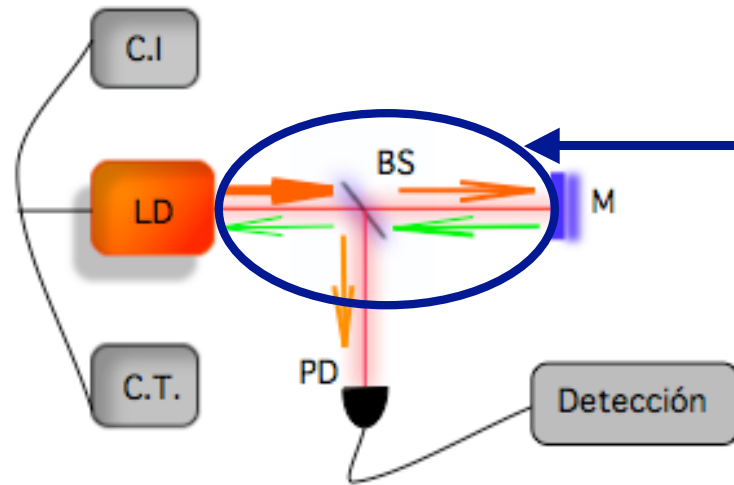
Optical feedback



Optical feedback



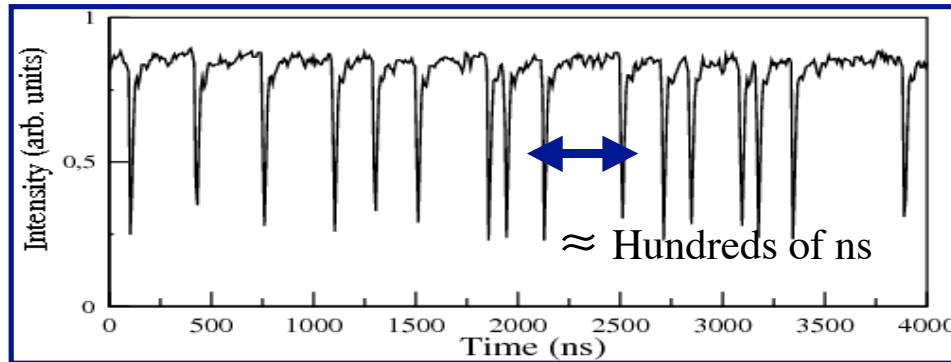
Optical feedback



Feedback time :
 $\tau_f = 2L/c$

Optical feedback

Weak to moderate feedback, and injection current close to threshold induces irregular fluctuations of the intensity.

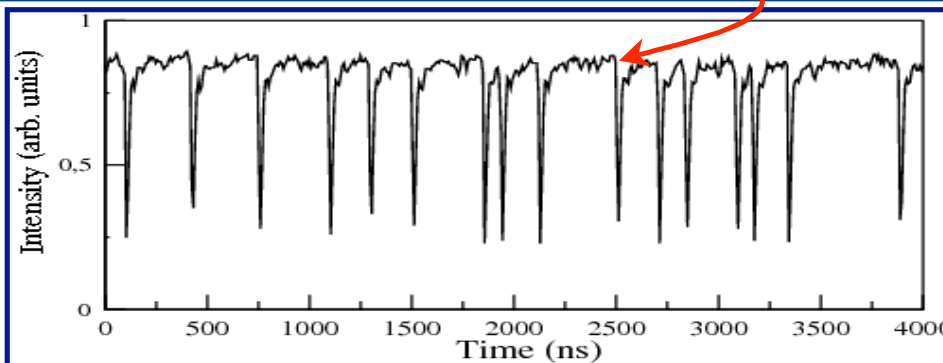
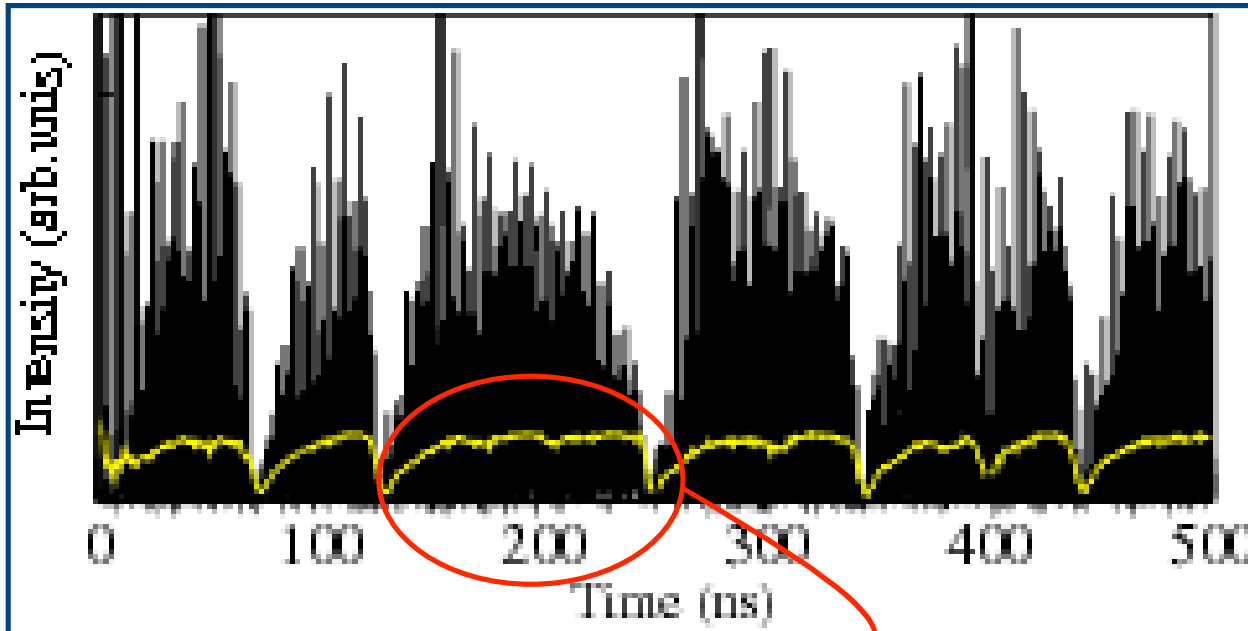


τ_f higher than the characteristics times of the system (relaxation oscillations) and moderate levels of feedback.

Space between dropouts higher than the relaxation oscillations frequency or the external cavity round-trip time (τ_f).

Low Frequency Fluctuations (LFF)

Optical feedback



Envelope of
the real signal
(order of ps)

Optical feedback

Lang and Kobayashi model, single mode, under weak feedback levels:

$$\frac{dE(t)}{dt} = \frac{1}{2}(1 + i\alpha) \left[G(t) - \gamma \right] E(t) + \sqrt{2\beta N} \xi(t) + \underline{\kappa_f e^{i\omega_0 \tau_f} E(t - \tau_f)}$$

$$\frac{dN(t)}{dt} = \frac{I}{e} - \gamma_e N - G(t) |E(t)|^2$$

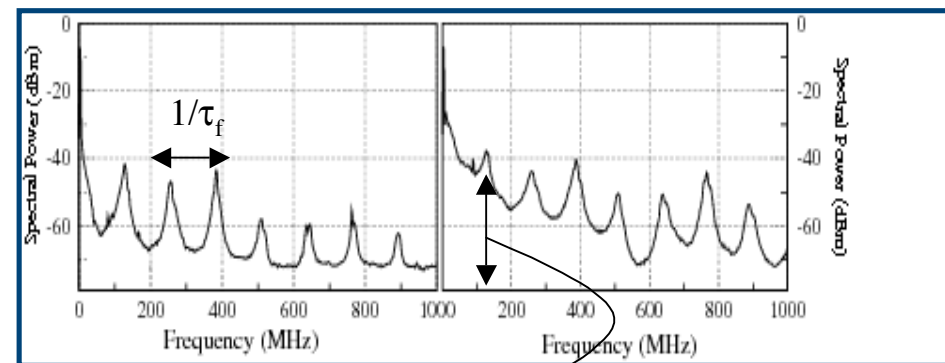
$$G(t) = \frac{g(N - N_t)}{1 + s |E|^2}$$

R. Lang and K. Kobayashi, IEEE JQE 16, 347 (1980)

Solutions:

External cavity modes (spaced $1/\tau_f$).

G.H.M van Tartwijk et al., IEEE JSTQE 1, 446 (1995)



LFF regime, higher spectrum for lower frequencies.

Optical injection

Injection of a light incoming from another laser.



We are focused on synchronized state .

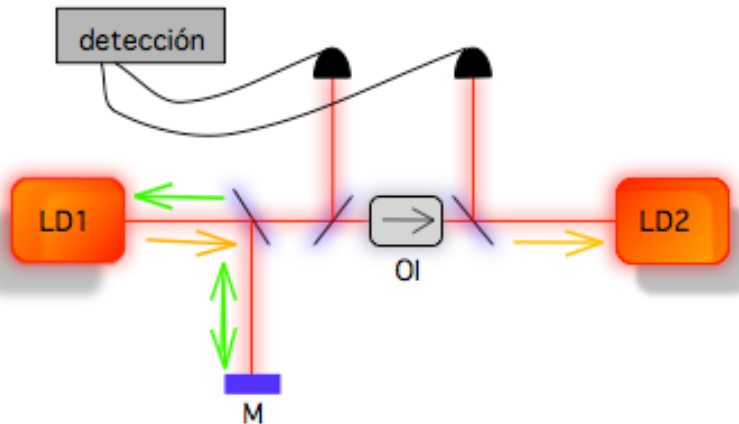
Adjustment of rhythms of oscillating objects due to their weak interaction.

- Lag synchronization: $x_1(t)=x_2(t-\tau)$.

The delay is determined by the fly time between the subsystems (τ_c)

Optical injection

Unidirectional coupling.



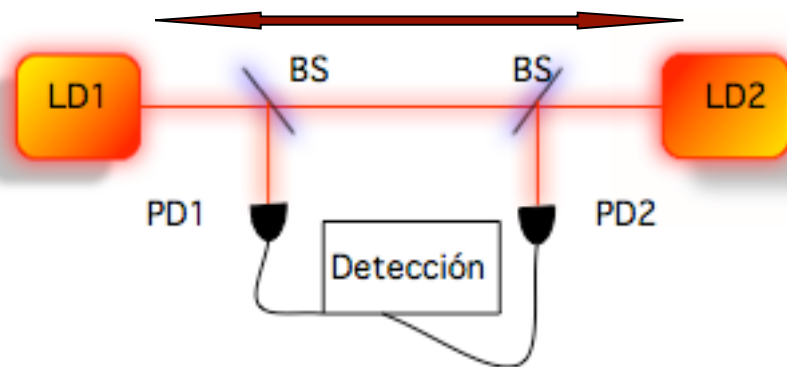
Chaos induced by the feedback in LD1 and transmitted to LD2.

LD2 synchronizes its output intensity with LD1 after a lag time τ_c .

Two characteristic times: feedback time τ_f and coupling time τ_c .

C. Masoller, PRL **86**, 2782 (2001)
A. Locquet et. al, PRE **65**, 056205 (2002)

Bidirectional coupling.



Natural generalization of the feedback system, replacing the mirror by another laser.

Delay time between synchronized signals: τ_c .

T. Heil et al., PRL **86**, 795 (2001)
J. Mulet et al., Proc. SPIE **4283**, 293 (2001)

Optical injection

Model based in Lang and Kobayashi equations:

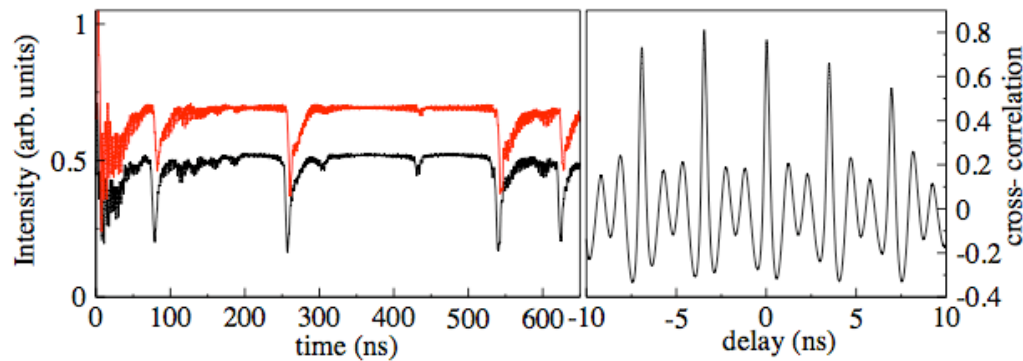
$$\frac{dE_j(t)}{dt} = \frac{1}{2}(1 + i\alpha_j) \left[G_j(t) - \gamma_j \right] E_j(t) + \sqrt{2\beta N_j} \xi_j(t) + \underbrace{\kappa_{3-j} e^{i(\Delta\omega t - \omega_{3-j}\tau_{c3-j})} E_{3-j}(t - \tau_{c3-j})}_{\text{Injection}} + \underbrace{\delta_{j1} \kappa_{f1} e^{-i\omega_1\tau_{f1}} E_1(t - \tau_{f1})}_{\text{Feedback}}$$

$$\frac{dN_j(t)}{dt} = \frac{I_j}{e} - \gamma_{ej} N_j - G_j(t) |E_j(t)|^2$$

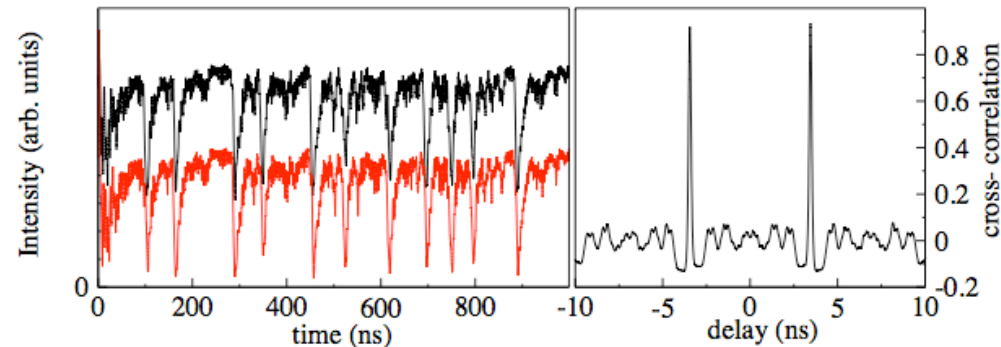
$$G_j(t) = \frac{g_j(N_j - N_{tj})}{1 + s_j |E_j|^2}$$

Optical injection Cross correlation for unfiltered signals

$$C(\Delta t) = \frac{\langle (P_1(t) - \langle P_1 \rangle)(P_2(t + \Delta t) - \langle P_2 \rangle) \rangle}{\sqrt{\langle (P_1(t) - \langle P_1 \rangle)^2 \rangle \langle (P_2(t) - \langle P_2 \rangle)^2 \rangle}}$$



Unidirectional: lag synchronization with a delay equal to the **coupling time**. The system shows a leader and a laggard in its dynamics.



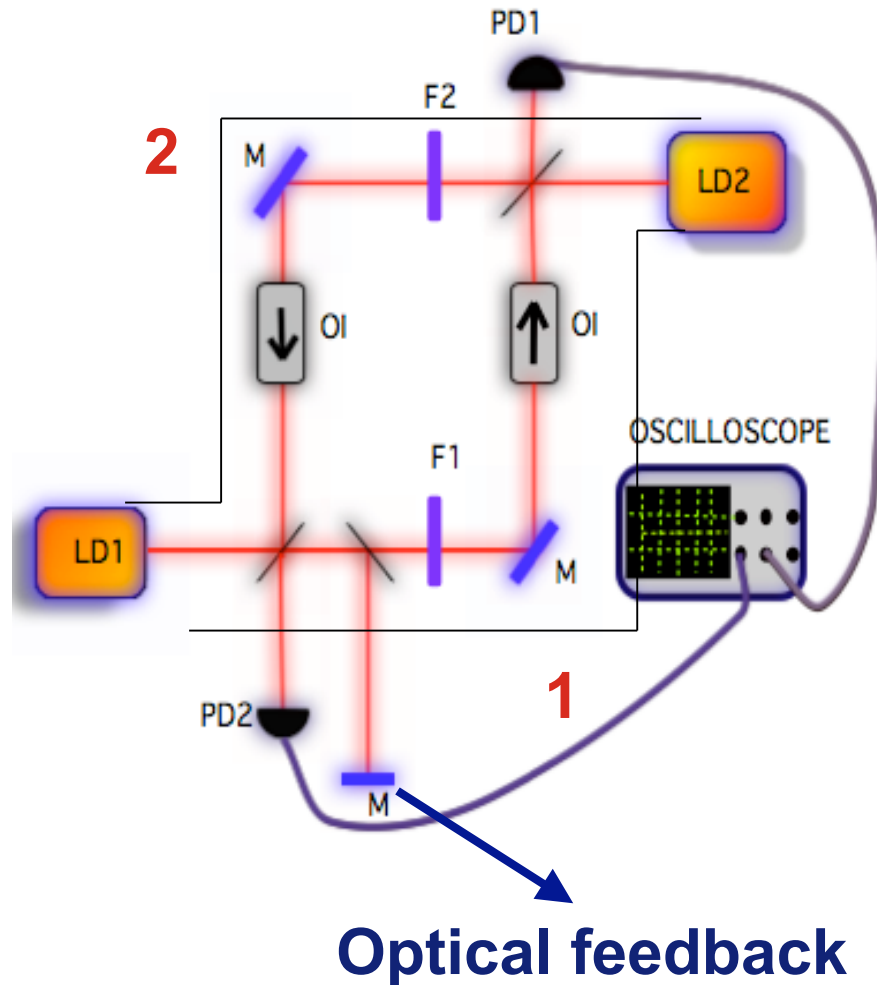
Bidirectional: lag synchronization with the leader and laggard roles **alternating** randomly between both lasers. If we include detuning between the lasers we determine the leader in the dynamics

J. Mulet et al., PRA **65**, 063815 (2002)

A.Hohl et al., PRL **78**, 4745 (1997)

T. Heil et al. PRL **86**, 795 (2001)

Control of leader-laggard dynamics



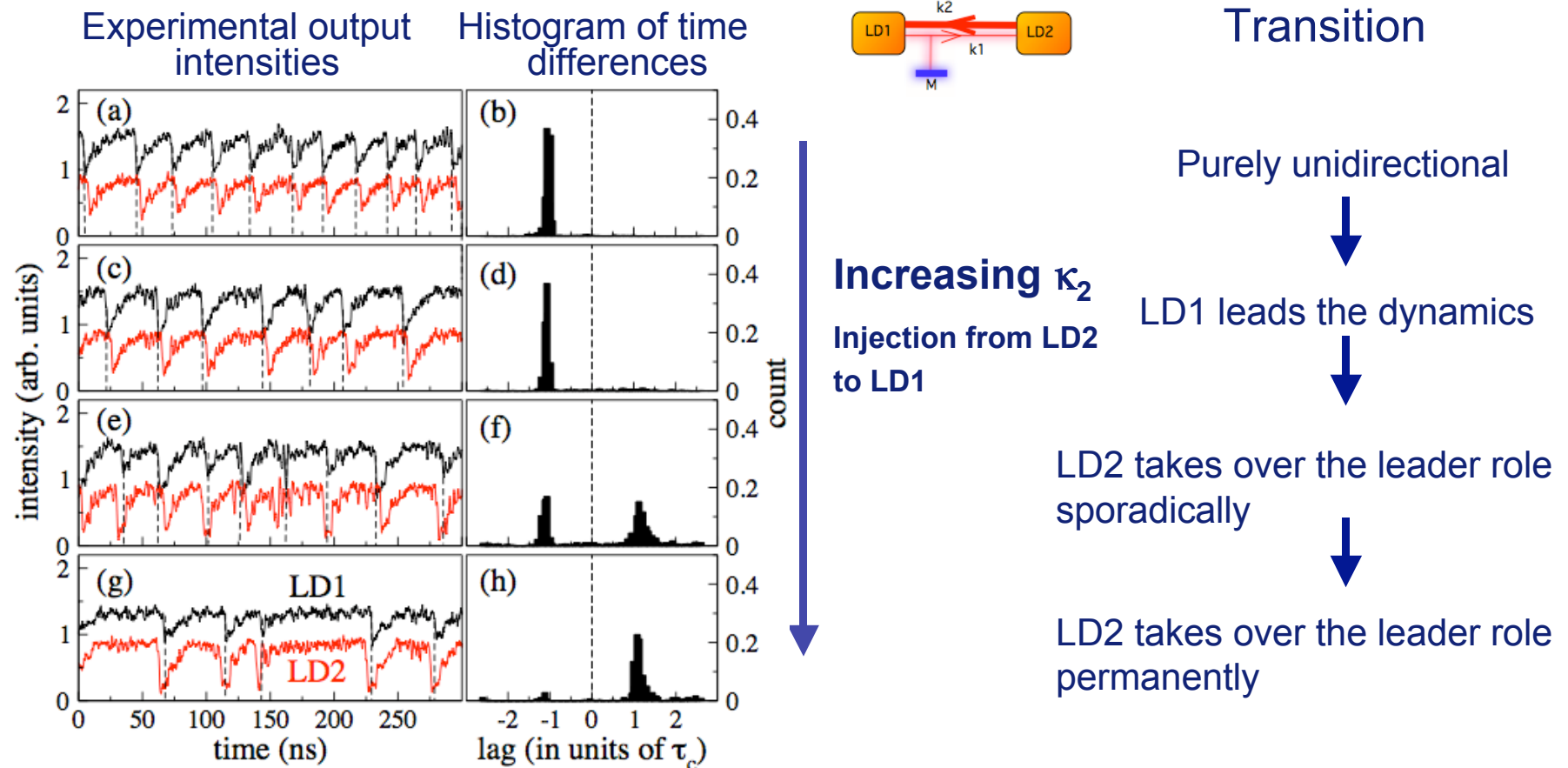
Two **mutually** coupled lasers through two independent unidirectional paths (**1** and **2**).

One of the lasers is subjected to optical feedback.

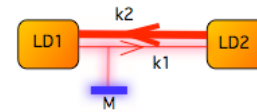
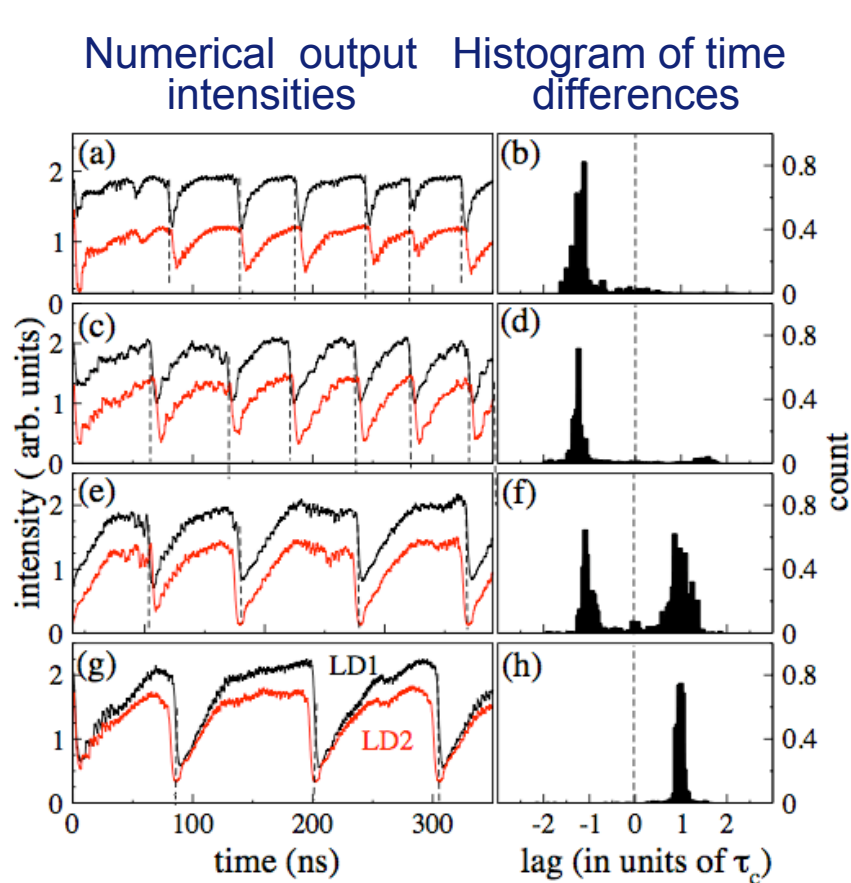
The amount of injection is controlled by two neutral density filters (F1 and F2)

Control of leader-laggard dynamics

Time lag determined by comparison between dropout events occurring in both lasers.



(a-b) 0% transmittivity, (c-d) 40%, (e-f) 63% and (g-h) 100%



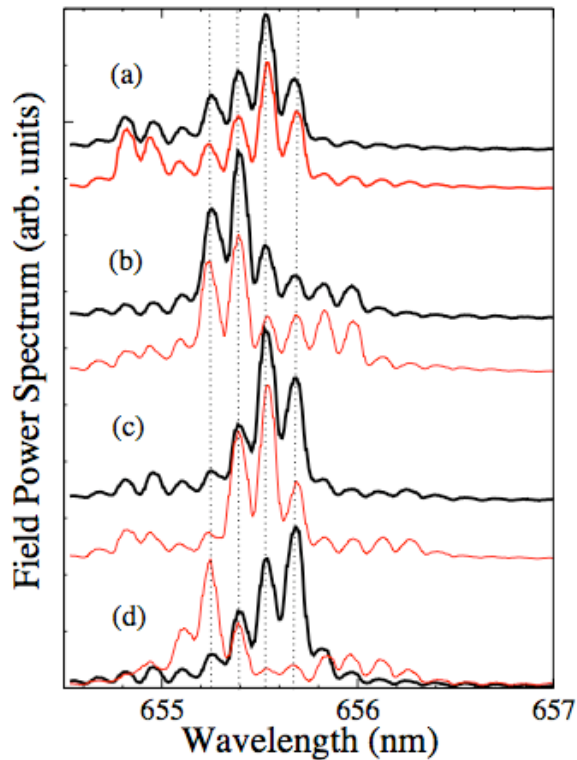
For a critical value a symmetric situation arises. The leader and laggard roles alternate randomly in time (as a symmetrical coupling).

Beyond this critical value LD2 dominates the dynamics.

Transition occurs for $k_2 > k_1$

For $k_1 = 80 \text{ ns}^{-1}$ and $k_f = 30 \text{ ns}^{-1}$ (a-b) $k_2 = 0$, (c-d) $k_2 = 50 \text{ ns}^{-1}$, (e-f) $k_2 = 70 \text{ ns}^{-1}$, (g-h) $k_2 = 90 \text{ ns}^{-1}$.

Field spectrums



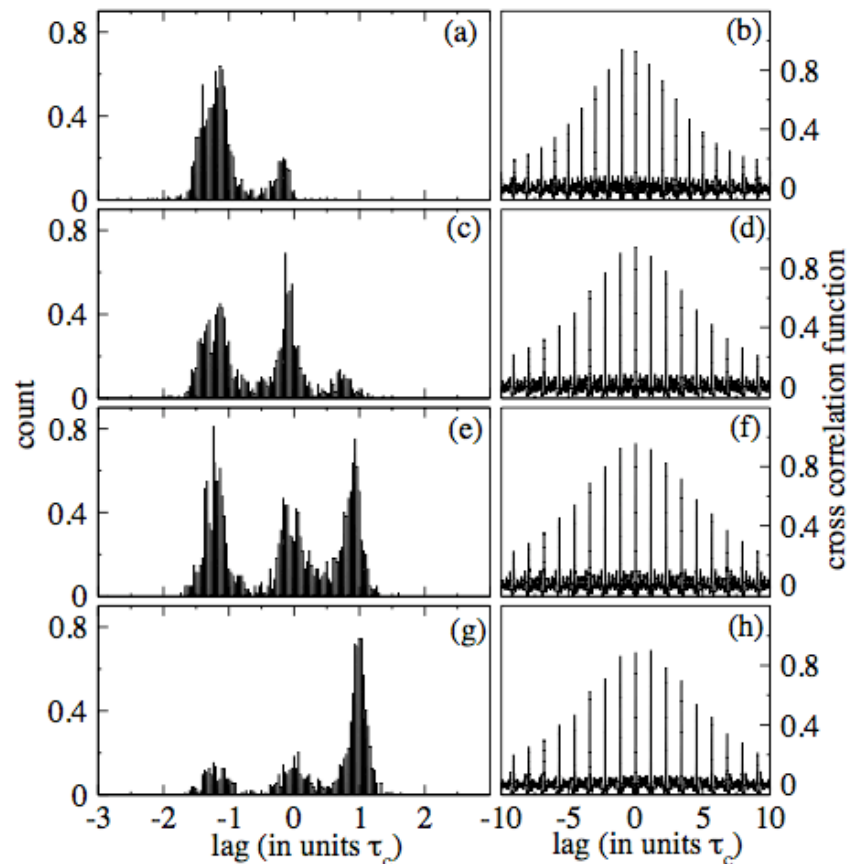
Due to the injection the LD2 wavelength travels towards higher values (a decrease in frequencies)

With high non symmetric interaction the spectrum locks, travelling together to lower frequencies.

As we increase the influence of LD2 in LD1 the wavelengths unlock, and after that starts the change in the leader of the dynamics of the system.

(a) 0% transmittivity, (b) 25%, (c) 32% and (d) 40% of F2.

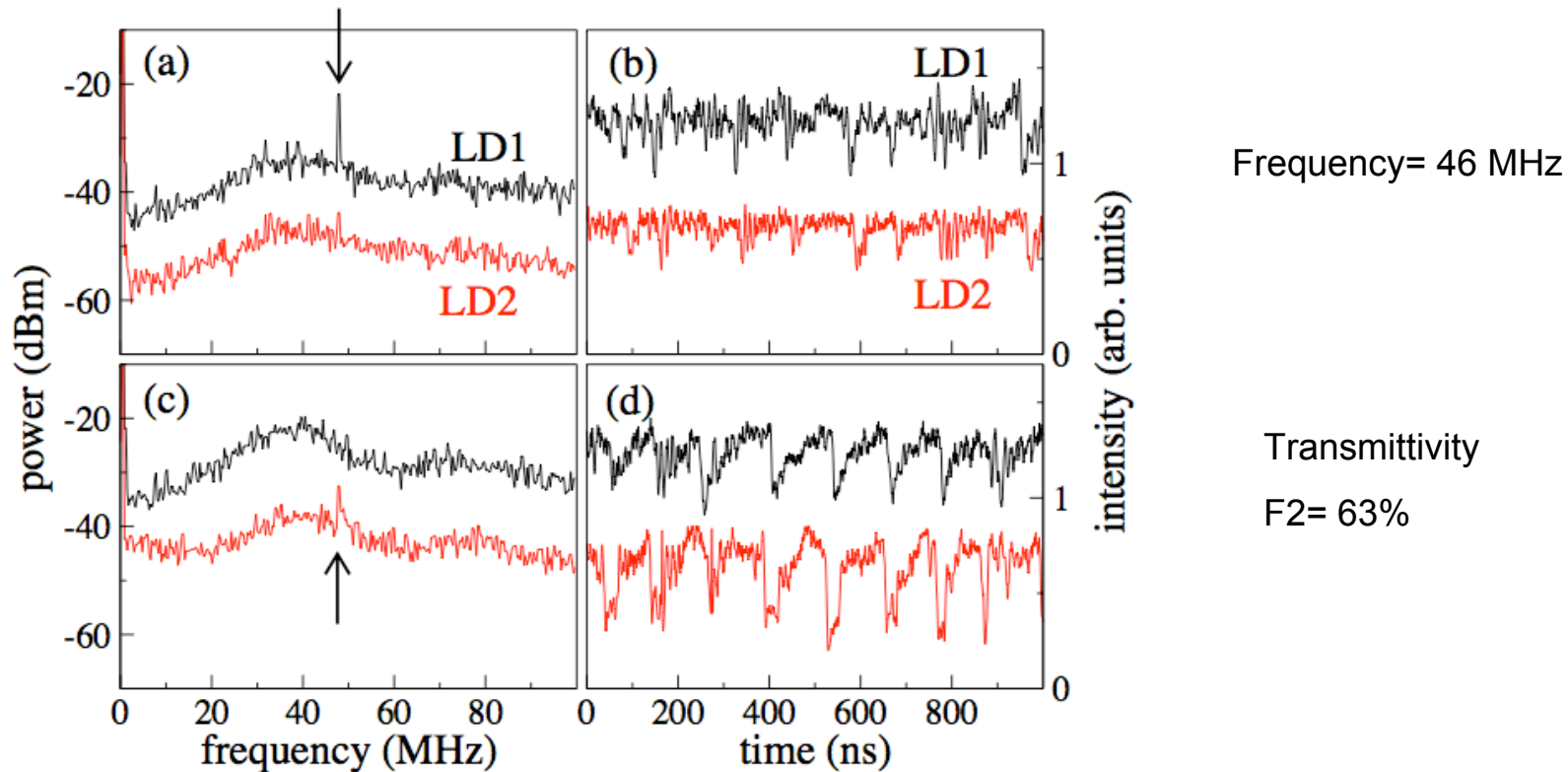
Quantification of the transition with comparison in fast time scales.



The system changes the leader, passing through a compound state of 0 lag and alternating the leader.

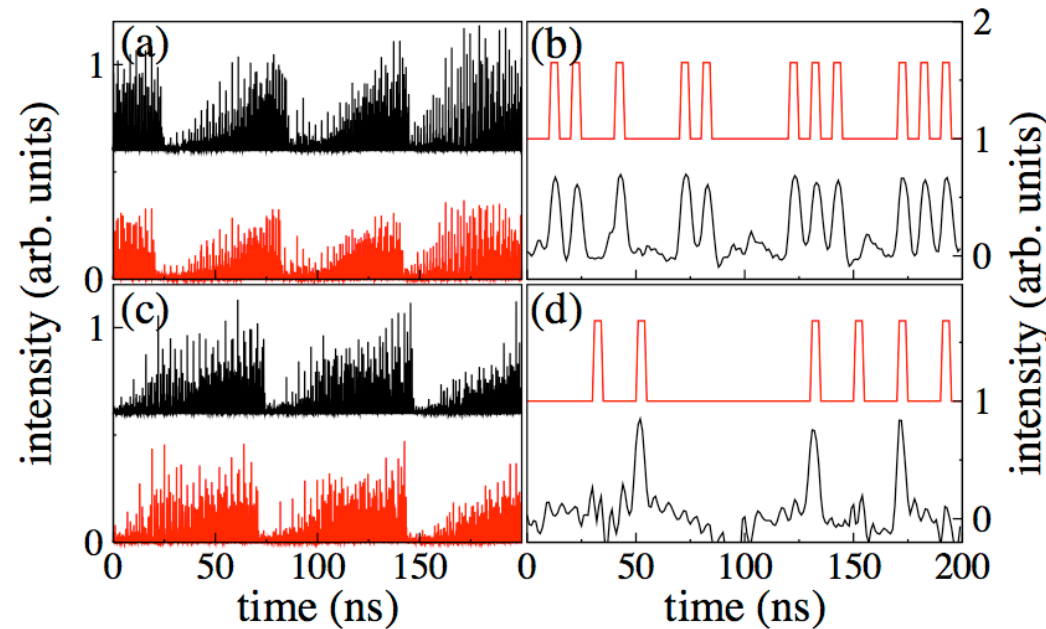
For $k_1 = 80 \text{ ns}^{-1}$ and $k_f = 30 \text{ ns}^{-1}$ (a-b) $k_2 = 60 \text{ ns}^{-1}$, (c-d) $k_2 = 65 \text{ ns}^{-1}$, (e-f) $k_2 = 70 \text{ ns}^{-1}$, and (g-h) $k_2 = 80 \text{ ns}^{-1}$.

Message transmission: filtering due to the synchronization of chaotic part of the signals (chaos-pass filtering) [T. Heil et. al. PRA **58**, R2672 (1998), I. Fischer et al., PRA **62**, 011801 (R)(2000)]



Control of leader-laggard dynamics

Introducing a bit message in the leader laser pumping current and we recover it in the receiver laser.

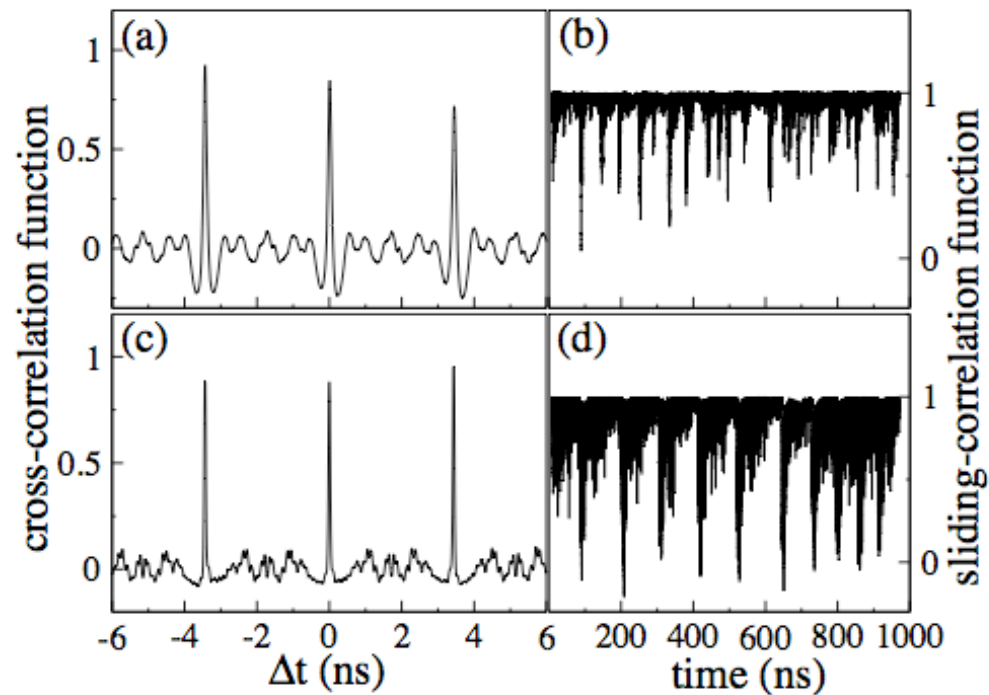


Leader LD1: correct recuperation.

Leader LD2: lose of information.

Output intensities without filterin, and introduced and recovered message for $k_1=80\text{ns}^{-1}$ y $k_f=30\text{ ns}^{-1}$ y (a)(b) LD1 leader, $k_2=0$ and (c)(d) LD2 leader, $k_2=90\text{ns}^{-1}$.

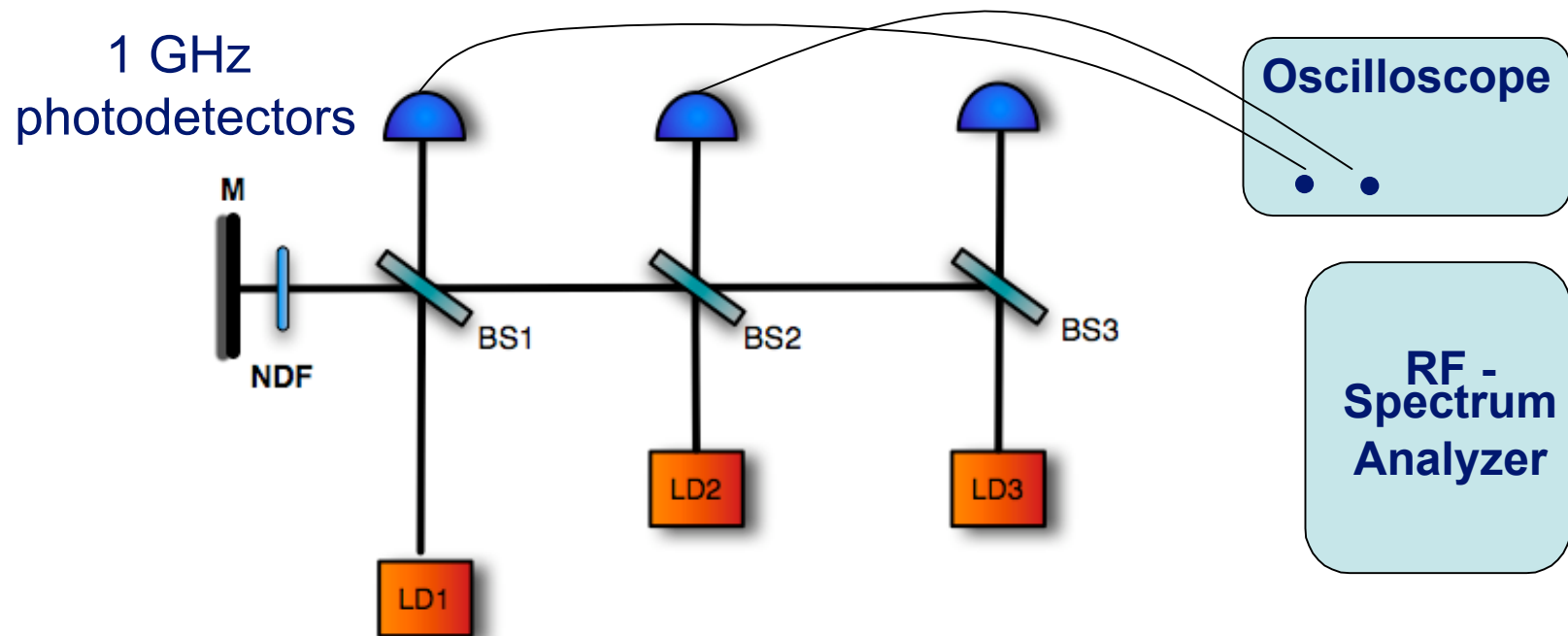
Slide correlation: maximum of correlation function computed with temporal averages over a moving time window [J. M. Buldú et al. , PRL 96, 024102 (2006)]



Cross correlation function and slide correlation for: (a)(b) LD1 leader, (c)(d) LD2 leader

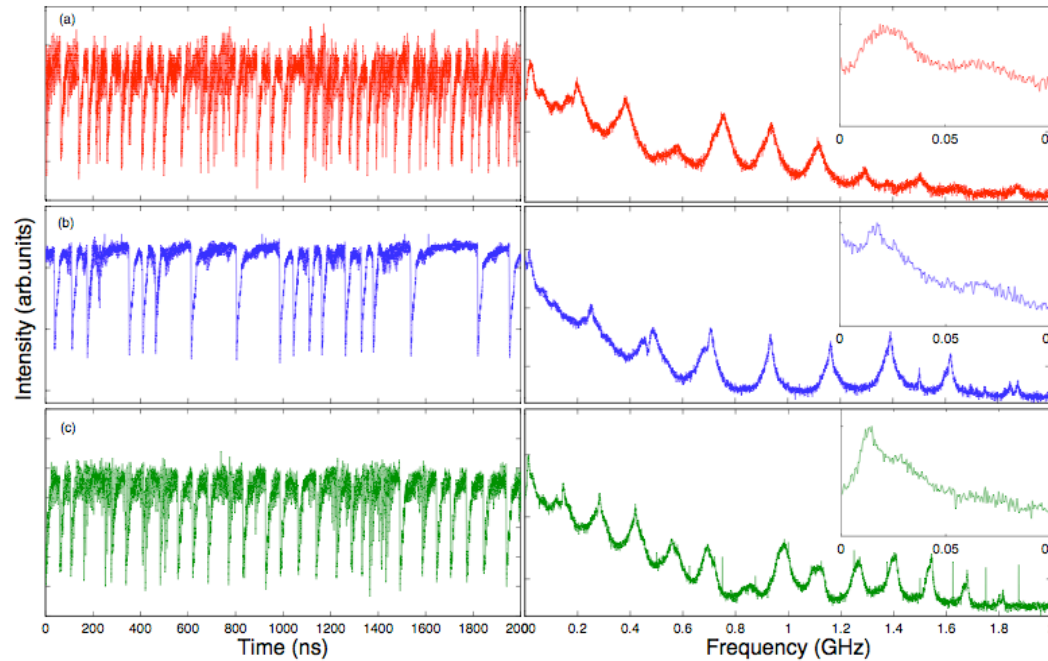
Synchronization via cluster formation

How the system loses its synchrony



Three AlGaInP index-guided and multiquantum well semiconductor lasers with feedback, mutually coupled through a mirror ($\lambda=650$ nm)

Laser intensities and RF spectra for uncoupled lasers.



Mean inter-dropout events

External cavity frequencies ($1/\tau_{ij}$)



Inset, low frequencies



Harmonics

Mean time between dropouts:

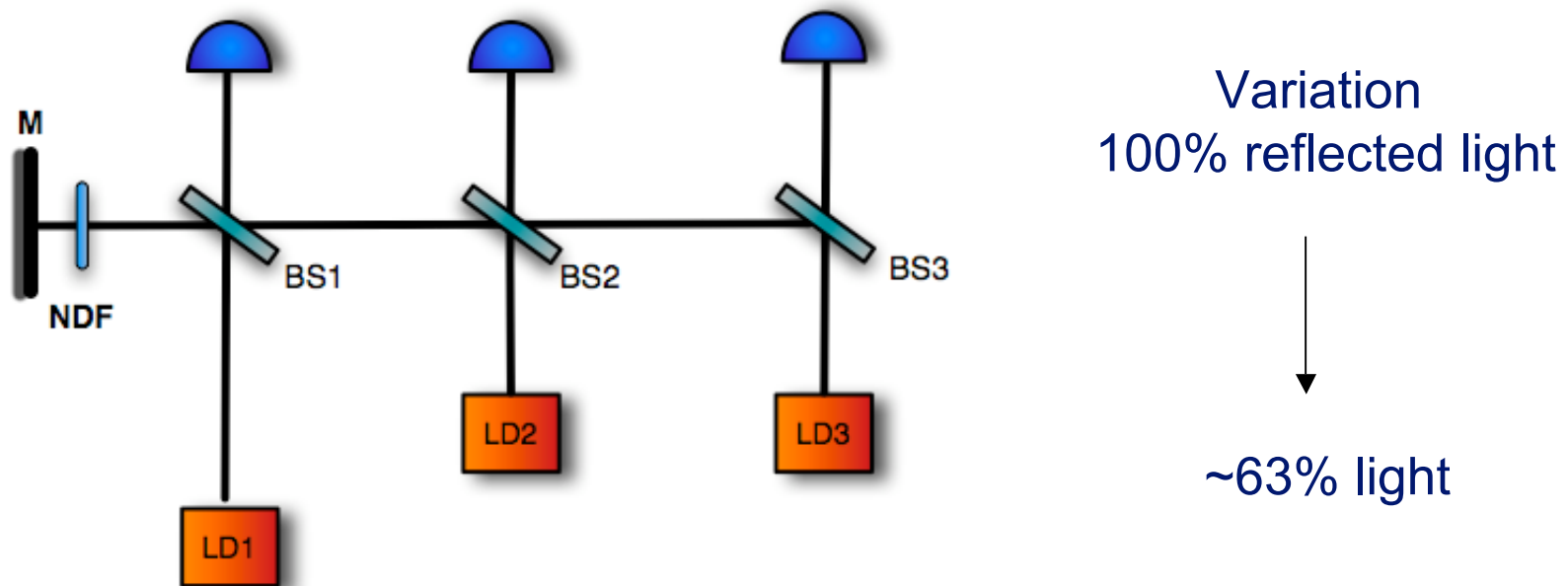
LD1= 200ns

LD2= 100 ns

LD3= 150 ns

Route to synchronization in a laser array

Injected light → NDF, neutral density filter



How the lasers lose their synchrony as the total injected light decreases

Synchronization

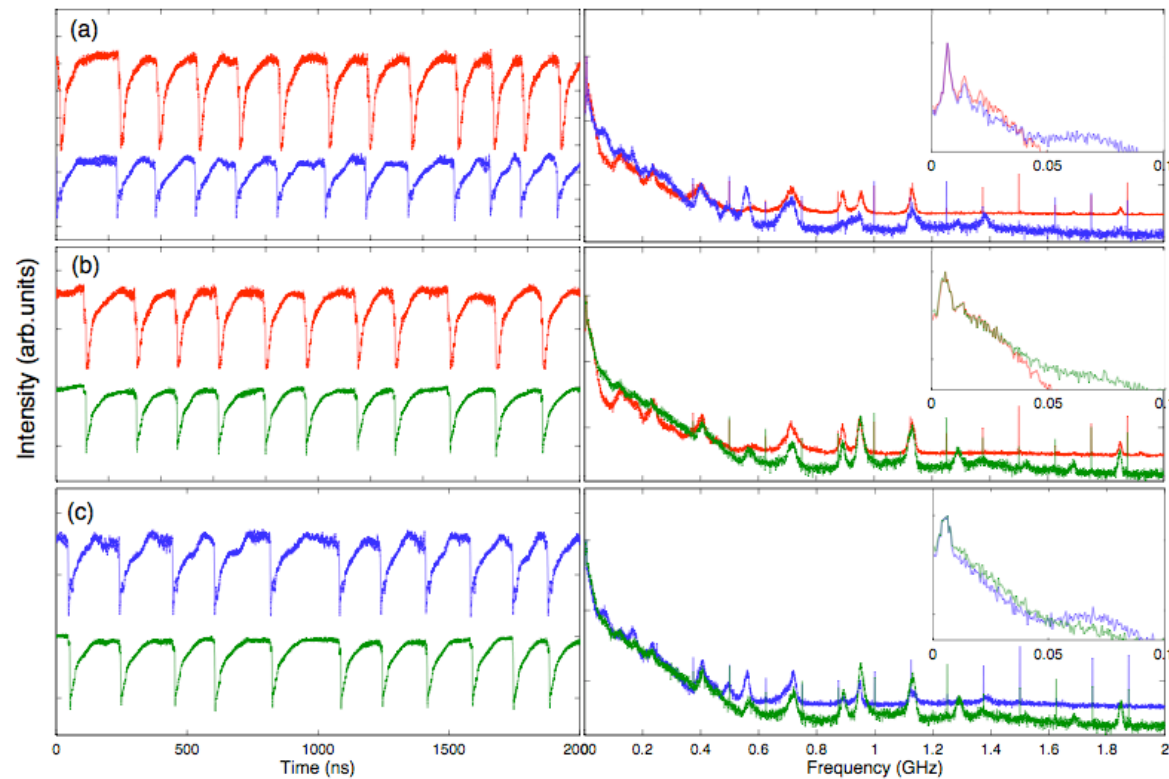
100% incoming light

lag times

Ld1-Ld2 \sim 10ns

Ld1-Ld3 \sim 0.5ns

Ld2-Ld3 \sim 10ns



LD1-LD2

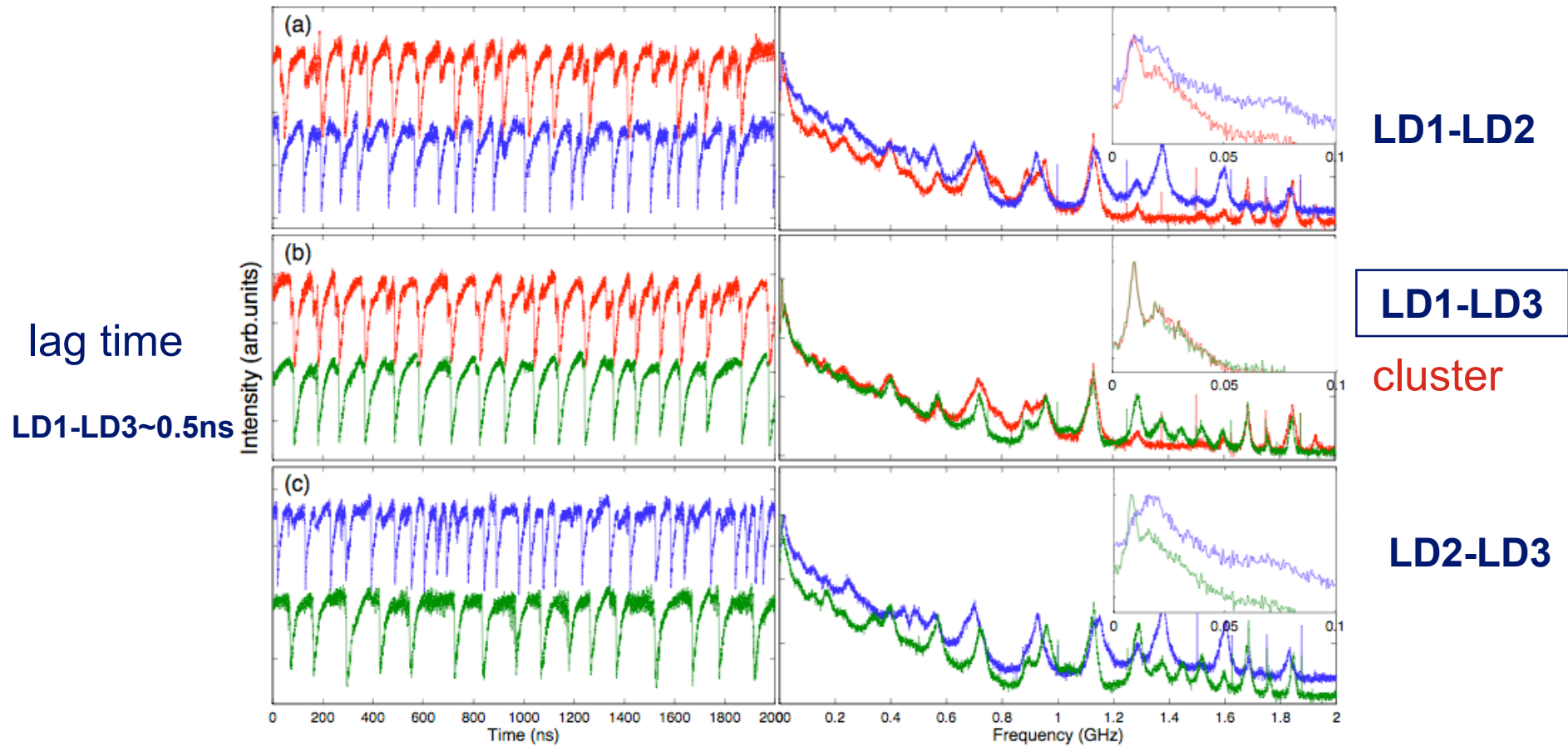
LD1-LD3

LD2-LD3

Mean time between dropouts \sim 165 ns

Clustering

NDF 63% Transmittivity

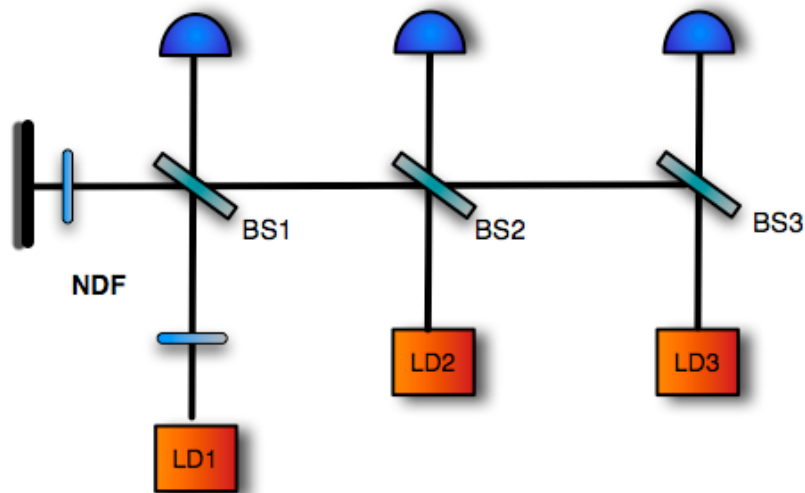


lag time

LD1-LD3~0.5ns

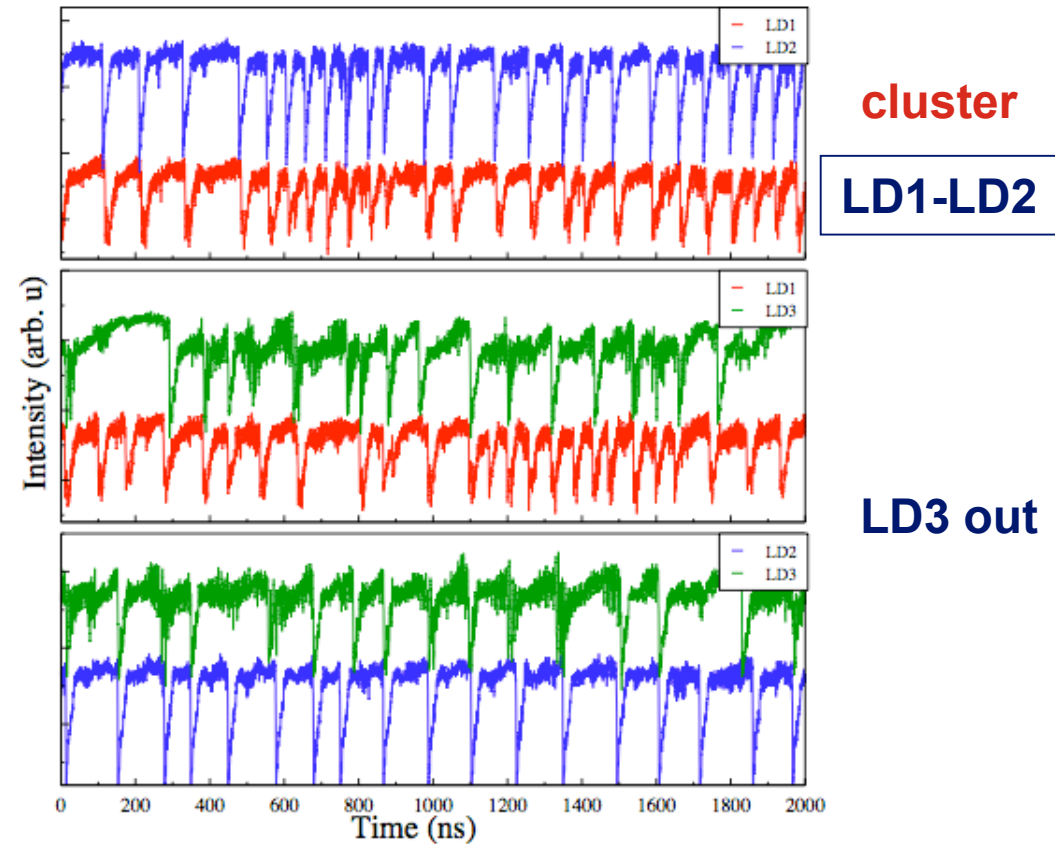
Mean time between dropouts ~100 ns

Change of lasers of the cluster



lag time LD1-LD2 ~ 5 ns.

Maintained in
synchronization state



Route to synchronization in a laser array

Rate equations for slowly-varying complex amplitude and the carrier density, in i th laser^[1]

$$\frac{dE_i}{dt} = i\omega_i E_i + \kappa(1 + i\alpha)E_i + \sqrt{D}\xi_i(t)$$

$$+ \sum_{j=1}^3 \eta_{ij} E_j(t - \tau_{ij}) e^{-i\omega_0 \tau}$$

$$\frac{dN_i}{dt} = \gamma_n (\mathbf{I} - N_i - N_i |E_i|^2)$$

ω_i : solitary frequency

ω_0 : reference common frequency.

$$\omega_0 = 2\pi c / \lambda_0$$

κ : cavity loss coefficient

D : spontaneous emission strength

α : linewidth enhancement factor

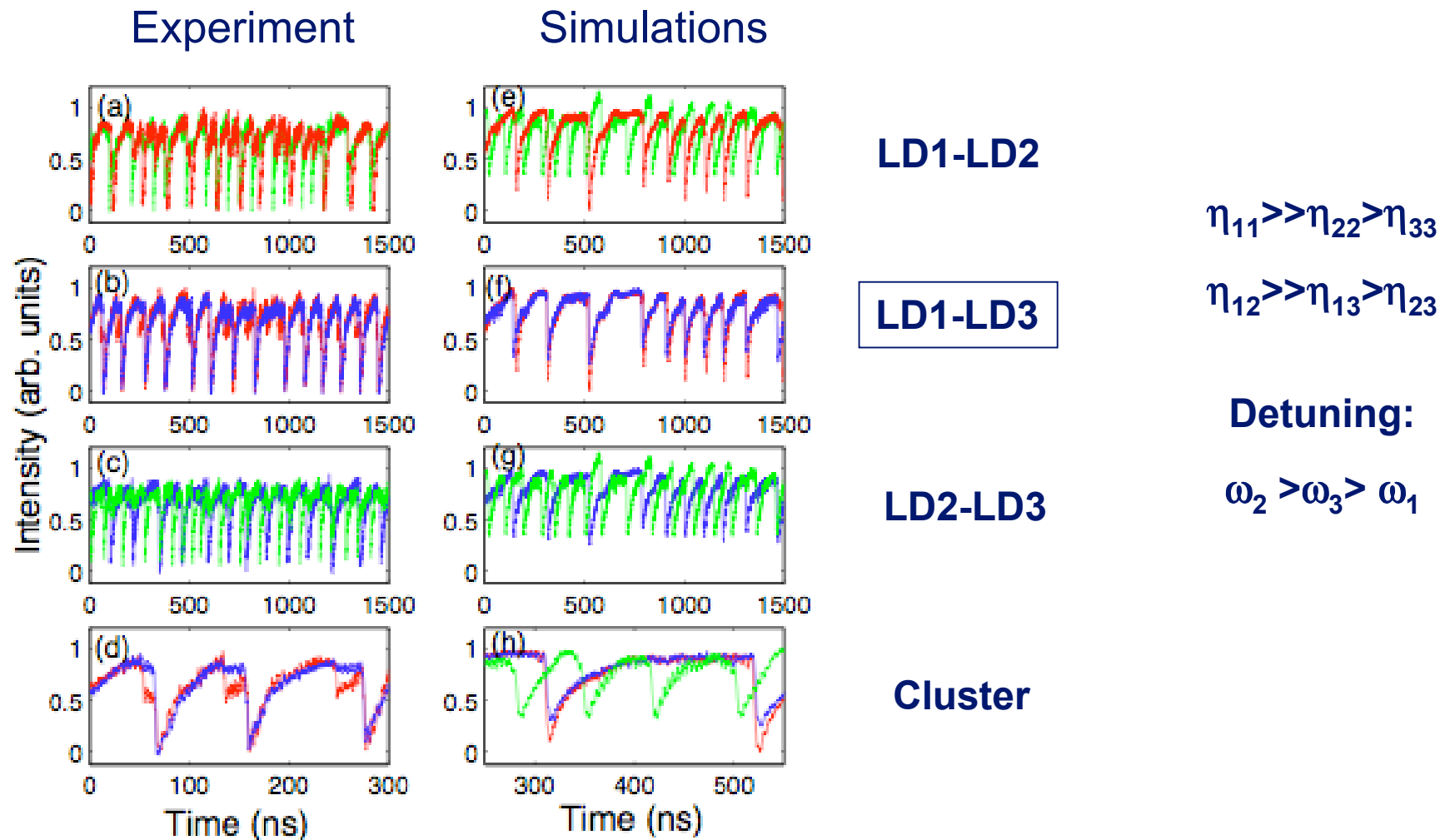
η_{ij} : coupling coefficients between
 Ld_i and Ld_j

[1] R. Lang, K. Kobayashi, *J. Quantum Electron* **16**,346 (1980);

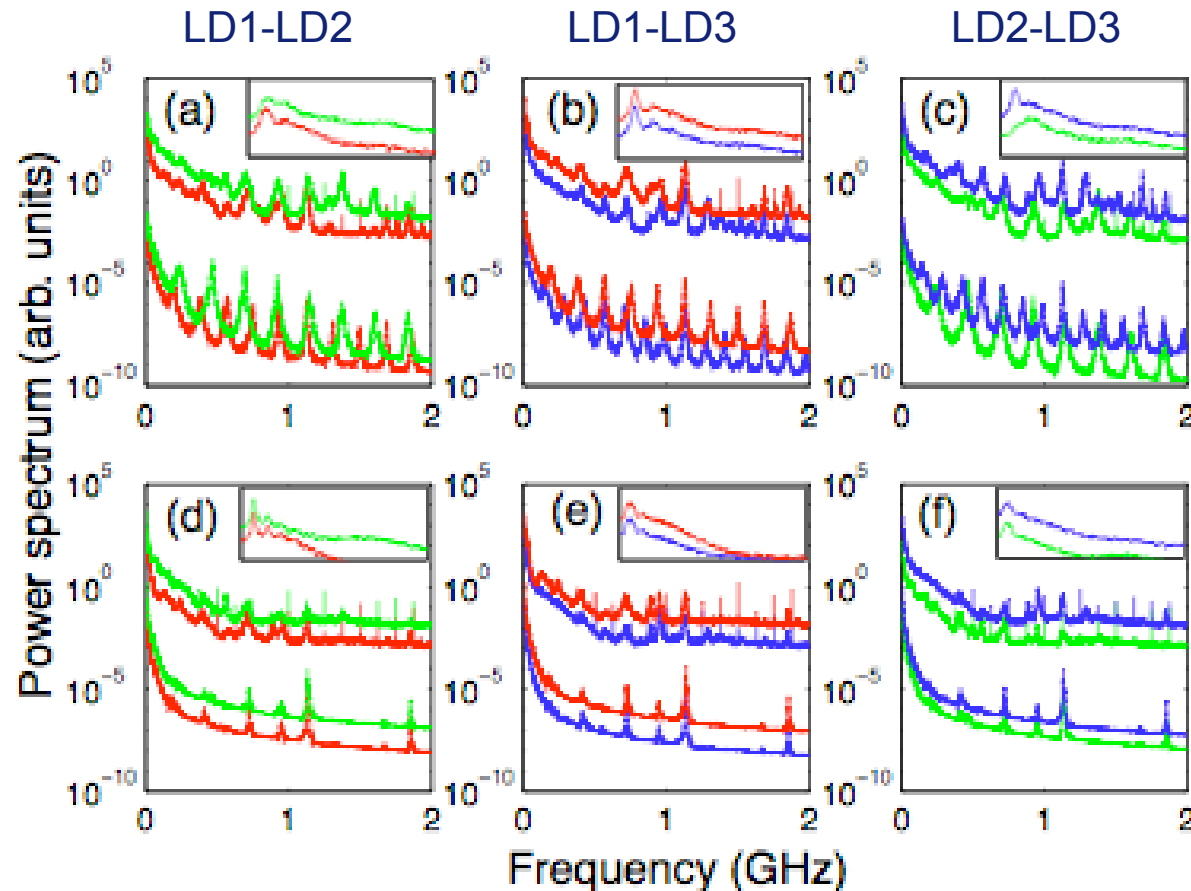
J. Garcia-Ojalvo, J. Casademont, M.C. Torrent, C.R. Mirasso, J.M. Sancho, *Int. J. Bif. Chaos* **9**,2225(1999),

G. Kozyreff, A. G. Vladimirov, P. Mandel, *Phys. Ref. Lett.* **18**, 3809 (2000)

Cluster formation



RF spectrum



Weak coupling

Cluster

Harmonics
begin to adjust

Strong coupling

Synchronization
Overlapping at low
and high frequencies

Conclusions

- Transition between unidirectional to bidirectional coupling occurs in our system through an alternating state.
- The switch of the leader in the dynamics in a coupled system is not a guarantee for bidirectional chaotic communications.
- Synchronization in a network of tree semiconductor lasers emerges with increasing coupling.
- On the route to synchronization, lasers cluster in pairs:
 - The dominant laser (LD1) has the strongest frequency shift due to the optical injection.
 - The third laser (LD2) needs an extra detuning (higher coupling strength) to become synchronized with the other two.