Numerical implementation of generalized Coddington equations for ophthalmic lens design.

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Abstract: A method for the numerical implementation of the generalized Coddington equations for ophthalmic lens design is presented. The tracing method is performed surface by surface and includes both finite ray tracing and generalized ray tracing methods. While finite ray tracing is used to provide the main direction of propagation of the considered ray, generalized ray tracing provides the principal curvatures of the wavefront and its orientation after being refracted by the lens. Three-dimensional representation of sagital and tangential powers is attained for all directions of gaze, and results are shown for some case studies. The validation of the proposed approach is double-checked using the spherical lens case.

1. Introduction

The different combinations of parameters that define the ophthalmic lens design cause different performance of the lens off-axis [1-3]. The classical theory of ophthalmic lens design proposes that the condition of compensation of the refractive error should be kept over the surface defined by the position of the remote point at all directions of gaze [2,4-6]. Although the aperture of the eye may be considered as small in general, this ideal condition is impossible to attain in practice, as only the curvature of one surface is available to the designer. For directions of gaze outside the optical axis, the image of an object blurs due to the presence of oblique astigmatism and the associated changes in effective power. The image of eccentric object points generates both a tangential and a sagittal focal line which change with the oblique angle in different ways (Fig.1).

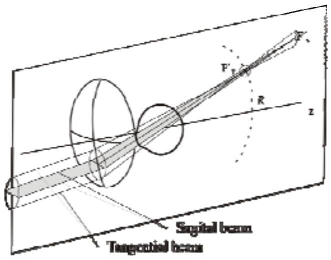

Figure 1. Oblique astigmatism in an off-axis object point for the lens-eye system; the different behavior in the sagital and tangential planes induces two different foci, creating oblique astigmatism

These foci can be obtained using the classical equations developed by Coddington in the early 19th Century [7]. In another approach, these foci can be obtained by wavefront tracing or generalized ray tracing. In this case the shape and orientation of the local wavefront in the vicinity of the principal ray becomes the parameter of interest.

We have developed a detailed numerical implementation of the equations proposed for ophthalmic lens design using the generalized ray tracing approach [8-11] under a Matlab environment. The implementation is described and some results are presented. The validation is performed using Beam4, a commercial ray tracing software, and Primer, a software for ophthalmic lens design based on the classical Coddington equations [12].

2. Finite ray tracing

When a ray arrives at a refracting surface, the directive vector of the refracted ray (***r'***) can be expressed as a linear combination of the incident ray vector (***r***) and the surface normal (***n***) as:

$$\boldsymbol{r}' = \mu \boldsymbol{r} + \gamma \boldsymbol{n} \tag{1}$$

This expression is the vectorial Snell law for refraction, with μ the ratio of refractive indexes and γ defined as:

$$\gamma = \mu(r \cdot n) + \left\{1 - \mu^2[1-(r \cdot n)^2]\right\}^{1/2} \tag{2}$$

3. Generalized ray tracing or wavefront tracing

Generalized ray tracing explains what happens to the principal directions and principal curvatures of the wavefront after propagation and refraction [8-12]. For propoagation, it can be shown that the principal directions of the wavefront are unchanged, and that the centers of curvature are fixed. For refraction, the curvatures of the refracted wavefront can be calculated using the generalized ray tracing equations which provide the refracted wavefront curvatures and the related torsion of the principal directions.

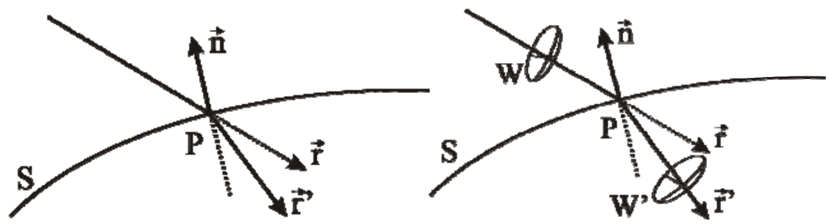

Figure 2. (a) Outline for ray tracing. A surface *S* separates two media of constant refractive index *n* and *n'*. An incident ray, with direction vector ***r***, intercepts the refracting surface at a point *P*, giving rise to a refracted ray whose direction vector is ***r'***. ***n*** is the normal to S at *P*. (b) Generalized ray tracing. The representation includes the incoming and outgoing wavefronts W and W'.

4. Ray tracing equations and wavefront equations for ophthalmic lens design

The evaluation method used in ophthalmic lens design considers that light behaves as if the eye had a fixed aperture with the dimension of the pupil size placed at the center of rotation of the eye, which are the classical approximations used in the field. This concept significantly simplifies the lens design problem, as the optical system of the eye is ignored, replacing it by a remote surface and an aperture at the center of rotation of the eye [1,2]. As mentioned, the method to evaluate the performance of the given ophthalmic lens includes both a finite ray trace and an ulterior generalized ray tracing procedure.

The position of the centers of curvature for the principal curvatures of the refracted wavefront associated with a particular ray will be interpreted as the sagital and tangential focal images. These directions of the focal images are determined by the principal directions of the refracted wavefront.

5. Results and validation

5.1. Calculated power distribution

The described equations have been implemented in a Matlab code allowing the calculation of the three-dimensional tangential and sagittal power distribution of an ophthalmic lens.

Figure 3 shows the 3D off axis tangential and sagital power distributions in one positive and one negative spherical ophthalmic lens with parameters described in the Figure footnote, used as examples. The differences in the off-axis behavior of both types of lenses, and the departure of the sagital and tangential cups in this simple case, are made evident. Difference sincrease for larger angles of gaze.

(a) (b)

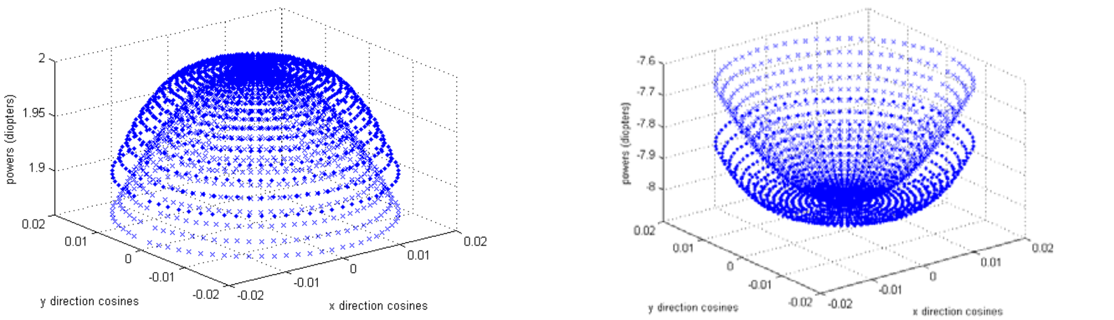

Figure 3. (a). 3D tangential (exterior cup) and sagital (interior cup) power distributions for a +2.00 D back vertex power lens with center thickness of 3mm, refractive index of 1.5, radius of the anterior surface 71,44 mm and radius of the posterior surface 98.05 mm. Center of rotation of the eye is set 27 mm behind the posterior surface. (b) 3D Tangential (exterior cup) and sagital (interior cup) power distributions for a -8.00 D back vertex power lens with central thickness of 1 mm, refractive index 1.7, radius of anterior surface 215.38 mm, and radius of the posterior surface 62.19 mm; the center of rotation of the eye is set 30 mm behind the posterior surface of the lens. Vertical axis plots power in diopters; x and y director cosines fix a particular direction of gaze in space.

5.2. Validation

The proposed code has been validated usng two different approaches. On one side, the slopes of the refracted rays after incidence in the first surface have been calculated, and the director cosines in the X and Y directions after refraction compared with those of a commercial raytracing software. For its ease of use, we used Beam4, from Stellar Software [13]. Figure 4b compares the two-dimensional power distributions of sagital and tangential powers using classical Coddington equations, and the ones obtained using generalized raytracing. It may be seen how both methods yield equivalent results in the particular configuration of coincidence.

(a) (b)

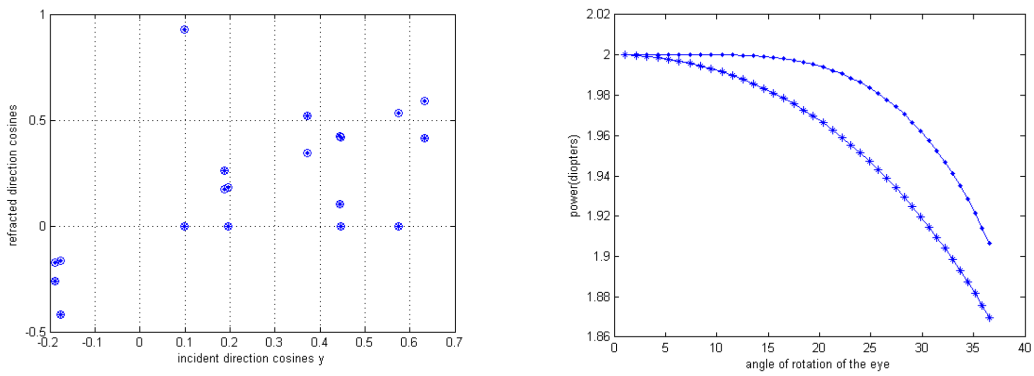

Figure 4. (a). Finite raytracing. Values in Matlab (circle) and in Beam 4 (*) for the X director cosines of the refracted ray on the anterior surface of the lens. In the same plot, values in Matlab (circle) and in Beam 4 (point) for the Y director cosines of the same ray and surface. (b) Generalized raytracing. Tangential and sagital powers (lines) in Matlab by generalized ray tracing, and tangential (.) and sagital powers (*) obtained by classical Coddington equations, for diferents directions of gaze for the +2.00D lens case. Coincidence is complete.

Conclusion

A generalized raytracing methodology has been implemented and programmed under a Matlab environment allowing calculation of 3D sagital and tangential power distributions in ophthalmic lenses. At present, the raytracing model is being extended to general surface shapes..

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