
#### Abstract

In this article the authors analyze the influence of the 0 number of samples in a training set on the accuracy of color and 1 spectral measurements made using a colorimetric and multispectral 2 imaging system. The authors develop a method for establishing the minimum and/or sufficient number of color samples in the training set above which the system's performance is independent of the number of samples. The authors also consider the dependence of the system's performance on the training set itself. Two setups of a charge coupled device camera-based imaging system are used for this purpose: a colorimetric configuration with three acquisition 9 channels and a multispectral configuration with seven acquisition channels. On the basis of the criterion established in this article, the 1 results show that the system's performance in terms of the accuracy of color measurement and spectral reconstruction seems to be in3 dependent of the training set used when over 110 samples are used 4 for the colorimetric configuration and over 120 samples for the mul5 tispectral configuration. This result is true for both the number of samples in the training set and the training set itself. The method that the authors developed can be generalized and implemented in 8 the industry for any application in which an imaging capture device is used for color and spectral measurements. © 2010 Society for Imaging Science and Technology. [DOI: XXXX]


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## NTRODUCTION

In recent years, industry has widely used commercially available standard digital imaging systems for color measurement. The popularity of these systems is mainly due to their ow cost and size, which means that they can be embedded in production lines. If an imaging system is to be used as a device for measuring color, its spectral sensitivities should be similar-or linearly related-to the color matching functions of the CIE standard observer. However, this is often not the case with conventional RGB color cameras, which are normally designed to achieve a good color appearance rather than high fidelity color reproduction. The results provided by standard imaging systems are therefore not as good as those obtained with specific colorimetric instruments or cameras with optimized spectral sensitivities. ${ }^{1}$ However, they can still be useful for industrial processes that require less color accuracy, such as in the automobile accessories industry and in the production of large format televisions and

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# Influence of the Number of Samples of the Training Set on Accuracy of Color Measurement and Spectral Reconstruction 

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printers. Moreover, multispectral imaging systems with more ${ }^{51}$ than three acquisition channels can be used to overcome 52 some of the limitations of conventional color systems, thus 53 allowing more accurate color measurements or even spectral 54 reconstructions. ${ }^{2}$ For this reason, it is of great importance to 55 characterize any imaging system colorimetrically or even 56 spectrally.

The colorimetric characterization of an imaging system 58 is the process of deriving the transformation that defines the 59 correspondence between the camera's digital responses and a 60 color space that is independent of the device, which may be 61 either XYZ or CIELAB. This process is essential to achieve 62 high fidelity color measurements. An initial approach for 63 this purpose is to use methods of colorimetric characteriza- 64 tion based on spectral sensitivities, ${ }^{3}$ which require the pre- 65 vious measurement of the system's spectral sensitivities. The 66 relationship between the camera's spectral sensitivities and 67 the CIE color matching functions must then be found so 68 that it can be used to transform the system's digital re- 69 sponses into $X Y Z$ values. These methods are usually only 70 applied to colorimetric configurations of imaging systems, 71 i.e., ones with three acquisition channels, due to the growing 72 complexity when the number of acquisition channels is in- 73 creased. Furthermore, although all the steps in this method 74 are very clear conceptually, their application involves several 75 fittings of experimental data, which leads to a considerable 76 amount of errors that can easily be accumulated in the esti- 77 mations of the XYZ values, particularly if the system's spec- 78 tral sensitivities are not the most suitable.

However, another approach can be used to transform a 80 conventional imaging system into an instrument for color 81 measurement or spectral reconstruction. It involves methods 82 that establish a direct relationship between the digital levels 83 of the response of the imaging system and the corresponding 84 tristimulus values or, equivalently, the reflectance spectra. ${ }^{4} 85$ These methods are faster and generally require a set of color 86 samples called the training set to train and characterize the 87 imaging system. In addition, they need another set of color 88 samples, called the test set, to test the system's characteriza- 89 tion and evaluate the accuracy of the subsequent estimation. 90 The validity of this alternative colorimetric characterization 91 is based on the fact that many industrial applications only 92
${ }^{93}$ require color measurements or spectral reconstructions of 94 certain color patterns, which usually have a rather limited 95 color gamut. Therefore, if we use a properly selected training 96 set that includes representative color samples similar to those 97 that will be later measured, this type of colorimetric calibra98 tion can provide good enough results.
99 In such cases, both the training and the test sets can be 100 made up by physical color charts or sets of color samples 101 that are specially selected or manufactured. There are no 102 universal training and test sets to characterize an imaging 103 system, although the GretagMacbeth ColorChecker DC 104 Chart (CCDC), ${ }^{5-8}$ and the GretagMacbeth ColorChecker 105 Color Rendition Chart (CCCR), ${ }^{6,9,10}$ the IT8 chart, ${ }^{8,11}$ the 106 color samples of the Munsell Book of Color, ${ }^{12,13}$ and the AQ: 107 color samples of the NCS (Ref. 14) are widely used. How\#1 108 ever, in many cases the training and test sets are selected 109 depending on the application of the imaging system: for 110 example, the color samples may be made from pigments 111 used in painting for restoration and preservation 112 applications ${ }^{5,6,15,16}$ or from natural objects. ${ }^{5,6,12,14,17,18}$
113 There are several methods and criteria for selecting the 114 color samples that constitute the training and test sets of an 115 imaging system. The aim is to generate a set of color samples 116 that are as diverse as possible so that the maximum number 117 of systems can be characterized with the minimum number 118 of color samples. ${ }^{2,4,14,19,20}$ The number of samples that are 119 needed in the training set to meet this objective depends on 120 the specific selection method used. However, the depen121 dence of the system's accuracy on the number of samples in 122 the training set, regardless of the selection method, has not 123 been thoroughly considered to date.
124 If we assume that the diversity of the color samples in 125 the training set is assured for all sizes, we can assume that 126 the greater the number of samples, the more accurate both 127 the color measurement and the spectral reconstruction. 128 However, it can be assumed that the increase in accuracy is 129 not unlimited and that there must be a minimum and/or 130 "sufficient number of samples" above which improvements 131 in accuracy are negligible or nil when the number of samples 132 is increased.
133 Regarding the influence of the number of color samples 134 in the training set on the accuracy of the estimation made by 135 the imaging system, previous studies have shown that, for a 136 trichromatic camera, the accuracy of the colorimetric esti137 mation (CIE XYZ tristimulus values) rose as the number of 138 color samples in the training set increased up to approxi139 mately 60 samples. There was no noticeable improvement in 140 training sets with over 60 samples. Hence, it can be estab141 lished that there is a kind of "limit" in accuracy improve142 ments brought about by increasing the number of color 143 samples in the training set. ${ }^{11,14,21}$ It appears that a training 144 set with between 40 and 60 color samples is needed to ob145 tain an estimate with reasonable accuracy. ${ }^{11,21}$ The mean er146 ror in color measurements made by spectral reconstruction 147 using training sets with a greater number of samples is prac148 tically independent of the size of the training set of the im149 aging system. ${ }^{14,21}$


Figure 1. Relative spectral sensitivities of the channels used in the colorimetric configuration of the imaging system (RGB tunable filter and CCD camera).


Figure 2. Transmittance spectra of the interference filters used in the multispectral configuration of the imaging system. Interference filters are named by their central wavelength.

In this article, we analyze the influence of the number of 150 samples in the training set on the system's accuracy in color 151 measurement and spectral reconstruction. We consider the 152 dependence of the system's performance on the size of the 153 training set and on the training set itself. Hence, we develop 154 a method for establishing the minimum and/or sufficient 155 number of color samples in the training set above which the 156 system's performance is independent of the number of 157 samples. Two configurations of a charge coupled device 158 (CCD) camera-based imaging system are used for this pur- 159 pose: a colorimetric configuration with three acquisition 160 channels and a multispectral configuration with seven acqui- 161 sition channels. ${ }^{22}$ This article is considered to be the first 162 step in the process of analyzing a system's accuracy in color 163 measurement and spectral reconstruction since it allows us 164 to define the training set used in any colorimetric or multi- 165 spectral imaging system, taking into account the accuracy 166 needed for the specific application. 167
The paper is structured as follows. The next section 168 describes the experimental setup and the configurations of 169 the imaging system, the methods used to perform the color 170 measurement and spectral reconstruction, and the selection 171 criterion applied to obtain a training set that has the greatest 172

Table I. Correspondence between the values of the inca and incb variables used in the selection criterion and the number of color samples selected from the CCDC chart.

| inca=incb | No. of color samples | inca=incb | No. of color samples |
| :---: | :---: | :---: | :---: |
| 17.00 | 10 | 2.15 | 100 |
| 9.20 | 20 | 1.69 | 110 |
| 6.98 | 30 | 1.45 | 120 |
| 5.35 | 40 | 1.22 | 130 |
| 4.50 | 50 | 0.80 | 140 |
| 4.00 | 60 | 0.58 | 150 |
| 3.12 | 70 | 0.20 | 160 |
| 2.68 | 80 | 0.05 | 166 |
| 2.36 | 90 |  |  |

173 possible variety of samples in terms of color ranges. The 174 results section presents a summary of the results obtained 175 for the colorimetric and multispectral configurations of the 176 CCD camera-based imaging system. Finally, the most rel177 evant conclusions are discussed in the last section.

## 178 MATERIAL AND METHOD

## 179 Experimental Setup and Configurations

180 We used an imaging system based on a QImaging QICAM 181 Fast 1394 monochrome 12-bit cooled CCD camera and a 182 Nikon AF Nikkor 28-105 mm objective lens. Two configu183 rations of this imaging system are considered: a colorimetric 184 configuration with three acquisition channels and a multi185 spectral configuration with seven acquisition channels. ${ }^{22}$
186 The colorimetric configuration is obtained by inserting 187 between the CCD camera and the objective lens a QImaging 188 RGB-HM-NS tunable filter, which is controlled through the 189 camera via software (Figure 1).
190 The multispectral configuration is obtained by inserting 191 between the CCD camera and the objective lens a motorized 192 filter wheel with seven CVI laser interference filters covering 193 the whole visible range of the spectrum and controlled via 194 software. The interference filters used have peak positions or 195 central wavelengths (CWLs) at 400, 450, 500, 550, 600, 650, 196 and 700 nm . All of them have full widths at half maximums 197 (FWHMs) of 40 nm , and their peak transmittances are $35 \%$, $19845 \%$, and $50 \%$, depending on the CWL (Figure 2).
199 The training sets considered in this study are made up 200 of a previous selection of color samples from the widely used 201 GretagMacbeth ColorChecker DC Chart (CCDC, 166 useful 202 color samples). The GretagMacbeth ColorChecker Color 203 Rendition Chart (CCCR, 24 color samples) is used as the 204 test set. These color charts are placed in a special light booth. 205 Incandescent lamps provide a uniform illumination field 206 over the charts when they are placed at the bottom of the 207 booth. The color samples are imaged using the two configu208 rations of the imaging system, and their reflectance spectra 209 are also measured using a tele-spectracolorimeter 210 PhotoResearch PR650 with an MS-75 objective lens.
211 Methods and Selection Criterion
212 The following methods are used for the color measurement 213 and spectral reconstruction from the system's digital re-


Figure 3. Colorimetric configuration: (a) mean CIELAB color difference and (b) mean RMSE values plotted vs the number of color samples in the training set (initial fixed color sample).
sponses: the pseudoinverse (PSE) method ${ }^{2,6,23-25}$ for the ${ }^{214}$ colorimetric configuration; and the principal component 215 analysis (PCA) method ${ }^{1,2,6,25,26}$ for the multispectral con- 216 figuration.

In the PSE method, a transformation matrix directly 218 relates the reflectance spectra of the color samples to the 219 corresponding digital responses of the imaging system. Us- 220 ing a training set of $N$ color samples with known reflectance 221 spectra, the transformation matrix $\left(D_{P S E}\right)$ is determined by 222 applying the Moore-Penrose pseudoinverse matrix as fol- 223 lows:

$$
\begin{align*}
& R_{(41 \times N)}=D_{P S E(41 \times k)} \cdot P(k)_{(k \times N)}  \tag{1}\\
& D_{P S E}=R \cdot P(k)^{t} \cdot\left[P(k) \cdot P(k)^{t}\right]^{-1} \tag{2}
\end{align*}
$$

where $R$ is the $41 \times N$ matrix of reflectance spectra for the $N 227$ color samples in the training set in which the reflectance 228 spectrum is sampled from 380 to 780 nm in intervals of 10229 nm , and $P(k)$ is the $k \times N$ matrix of the digital responses of 230 the $k$ acquisition channels of the imaging system for the $N 231$ color samples in the training set.

This transformation matrix allows the reflectance spec- 233 trum of any color sample to be estimated from its digital 234

Table II. Colorimetric contiguration: mean, minimum, maximum, and standard deviation of the CIELAB color difference and the RMSE values obtained vsing different sized iraining sets (inititil fixed color sample) from the CCDC chart. The CCCR chart was used as the test set.

| No. of color samples | Mean $\Delta E_{\text {ab }}{ }_{\text {b }}$ | $\min \Delta E_{\text {ab }}^{*}$ | $\max \Delta E_{\text {ab }}^{*}$ | Std. dev $\Delta E_{\text {ab }}^{*}$ | Mean RMSE | Min RMSE | Max RMSE | Std. dev RMSE |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 12.751 | 2.179 | 90.215 | 19.211 | $7.905 \times 10^{-2}$ | $2.918 \times 10^{-2}$ | $1.989 \times 10^{-1}$ | $4.352 \times 10^{-2}$ |
| 20 | 5.606 | 0.295 | 12.216 | 3.278 | $6.086 \times 10^{-2}$ | $2.647 \times 10^{-2}$ | $1.671 \times 10^{-1}$ | $3.201 \times 10^{-2}$ |
| 30 | 5.317 | 1.777 | 13.503 | 3.356 | $5.524 \times 10^{-2}$ | $3.307 \times 10^{-2}$ | $1.688 \times 10^{-1}$ | $2.802 \times 10^{-2}$ |
| 40 | 5.885 | 2.577 | 14.097 | 3.358 | $5.779 \times 10^{-2}$ | $3.405 \times 10^{-2}$ | $1.746 \times 10^{-1}$ | $3.053 \times 10^{-2}$ |
| 50 | 5.852 | 1.941 | 14.118 | 3.511 | $5.717 \times 10^{-2}$ | $2.923 \times 10^{-2}$ | $1.742 \times 10^{-1}$ | $3.055 \times 10^{-2}$ |
| 60 | 5.763 | 2.198 | 14.110 | 3.460 | $5.607 \times 10^{-2}$ | $2.762 \times 10^{-2}$ | $1.735 \times 10^{-1}$ | $3.030 \times 10^{-2}$ |
| 70 | 5.903 | 2.256 | 14.148 | 3.426 | $5.616 \times 10^{-2}$ | $3.075 \times 10^{-2}$ | $1.732 \times 10^{-1}$ | $2.989 \times 10^{-2}$ |
| 80 | 5.681 | 2.323 | 13.967 | 3.431 | $5.569 \times 10^{-2}$ | $2.983 \times 10^{-2}$ | $1.720 \times 10^{-1}$ | $2.950 \times 10^{-2}$ |
| 90 | 5.710 | 2.291 | 13.891 | 3.411 | $5.561 \times 10^{-2}$ | $3.035 \times 10^{-2}$ | $1.711 \times 10^{-1}$ | $2.898 \times 10^{-2}$ |
| 100 | 5.537 | 2.089 | 13.719 | 3.393 | $5.534 \times 10^{-2}$ | $3.082 \times 10^{-2}$ | $1.717 \times 10^{-1}$ | $2.929 \times 10^{-2}$ |
| 110 | 5.628 | 2.225 | 13.794 | 3.333 | $5.569 \times 10^{-2}$ | $3.213 \times 10^{-2}$ | $1.706 \times 10^{-1}$ | $2.872 \times 10^{-2}$ |
| 120 | 5.625 | 2.265 | 13.837 | 3.368 | $5.556 \times 10^{-2}$ | $2.987 \times 10^{-2}$ | $1.715 \times 10^{-1}$ | $2.933 \times 10^{-2}$ |
| 130 | 5.735 | 2.312 | 13.977 | 3.415 | $5.586 \times 10^{-2}$ | $2.967 \times 10^{-2}$ | $1.716 \times 10^{-1}$ | $2.945 \times 10^{-2}$ |
| 140 | 5.726 | 2.255 | 13.858 | 3.409 | $5.597 \times 10^{-2}$ | $3.046 \times 10^{-2}$ | $1.710 \times 10^{-1}$ | $2.911 \times 10^{-2}$ |
| 150 | 5.749 | 2.218 | 13.879 | 3.404 | $5.622 \times 10^{-2}$ | $3.152 \times 10^{-2}$ | $1.716 \times 10^{-1}$ | $2.928 \times 10^{-2}$ |
| 160 | 5.777 | 2.219 | 13.905 | 3.410 | $5.633 \times 10^{-2}$ | $3.118 \times 10^{-2}$ | $1.723 \times 10^{-1}$ | $2.963 \times 10^{-2}$ |
| 166 | 5.886 | 2.241 | 13.962 | 3.414 | $5.658 \times 10^{-2}$ | $3.215 \times 10^{-2}$ | $1.728 \times 10^{-1}$ | $2.972 \times 10^{-2}$ |

${ }^{235}$ responses provided that the training set is a good represen236 tation of all the color samples that will be measured with the 237 imaging system.
238 In the PCA method, a previous principal component 239 analysis is applied to the matrix of reflectance spectra for the 240 color samples in the training set to obtain the eigenvector 241 basis and the coefficients of the reflectance spectra on this 242 basis:

243

$$
\begin{equation*}
R_{(41 \times N)}=V_{(41 \times p)} \cdot C_{(p \times N)} \tag{3}
\end{equation*}
$$

244 where $V$ is the $41 \times p$ matrix for the $p$ first eigenvectors, and $245 C$ is the $p \times N$ matrix of scalar coefficients of the reflectance 246 spectra for the $N$ color samples in the training set on the 247 eigenvector basis $V$.
248 The scalar coefficients of the reflectance spectra of the $N$ 249 color samples in the training set are related to the digital 250 responses of the imaging system by means of a transforma251 tion matrix $\left(D_{P C A}\right)$ that is determined by applying the 252 Moore-Penrose pseudoinverse matrix:

253

$$
\begin{equation*}
C_{(p \times N)}=D_{P C A(p \times k)} \cdot P(k)_{(k \times N)}, \tag{4}
\end{equation*}
$$

254

$$
\begin{equation*}
D_{P C A}=C \cdot P(k)^{t} \cdot\left[P(k) \cdot P(k)^{t}\right]^{-1} . \tag{5}
\end{equation*}
$$

255 This transformation matrix allows the coefficients of any 256 color sample to be calculated on the eigenvector basis from 257 the digital responses of the imaging system. The linear com258 bination of the eigenvectors with the calculated coefficients 259 provides an estimation of the reflectance spectrum of the
color sample. From the spectral reflectances estimated using 260 the PSE and PCA methods, the CIELAB coordinates of each 261 color sample can be easily computed.

262
The selection criterion applied in this article to obtain a 263 training set with the greatest variety of samples possible, in 264 terms of color ranges, for each number of samples consid- 265 ered is based on differences in the $a^{*}\left(\Delta a^{*}\right)$ and $b^{*}\left(\Delta b^{*}\right) 266$ CIELAB coordinates between each pair of color samples in 267 the final selected set. Each pair of selected samples must 268 fulfill:

269

$$
\begin{equation*}
\Delta a^{*} \geq i n c a \quad \text { and } \quad \Delta b^{*} \geq i n c b \tag{6}
\end{equation*}
$$

where the values for the inca and incb variables are chosen 271 so that inca=incb for simplicity, and allow us to establish 272 the number of color samples to be selected.

273
The two configurations of the imaging system are 274 trained using training sets of sizes between 10 and 166 color 275 samples in steps of ten samples (Table I), and their perfor- 276 mance is tested using the 24 color samples from the CCCR 277 chart. The accuracy of the color measurement is evaluated in 278 terms of the mean, minimum, maximum, and standard de- 279 viation of the CIELAB color difference between the esti- 280 mated and the measured tristimulus values of the CCCR 281 color samples. The accuracy of the spectral reconstruction is 282 evaluated in terms of the mean, minimum, maximum, and 283 standard deviation of the root mean square error (RMSE) 284 between the reconstructed and the measured reflectance 285 spectra of the CCCR color samples.
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Figure 4. Colorimetric configuration: (a) mean CIELAB color difference and (b) mean RMSE values plotted versus the number of color samples in the training set for the five initial randomly selected color samples and the resulting training sets.

We carry out two analyses of the training set selection 288 process. In the first analysis, the first useful non-neutral 289 color sample from the CCDC chart (color sample B2) is 290 used as the initial fixed color sample. Starting from this 291 sample, training sets of different sizes are selected from the 292 CCDC color samples by applying the selection criterion row 293 by row. That is, the first pair of color samples to be com294 pared is B2 and C2. If they fulfill $\Delta a^{*} \geq$ inca and $\Delta b^{*}$ $295 \geq$ incb, C2 is incorporated into the training set and com296 pared with the next available patch, in this case, D2. If the 297 color samples do not meet the imposed condition, sample 298 C 2 is rejected and B2 is again compared with the next avail299 able patch (D2). This process continues until the necessary 300 number of color samples is reached for each size of the 301 training set. Therefore, the selection of the patches depends 302 strongly on the initial fixed color sample. With this first 303 analysis, the results will show that the performance of the 304 imaging system depends on the size of the training set but 305 not on the training set itself, since the training sets that are 306 selected will be specific to the initial fixed color sample and 307 mainly of lower sizes. To overcome this limitation and ana308 lyze the dependence of the system's performance on different 309 sized training sets, a second analysis is performed. In this 310 second analysis, an initial color sample is selected randomly


Figure 5. Colorimetric configuration: (a) mean CIELAB color difference values and (b) mean RMSE values obtained using the five training sets generated from an initial randomly selected CCDC color sample, for all sizes of the training set considered. CCCR color samples were used as the test set.
from the CCDC chart (initial randomly selected color ${ }^{311}$ sample). Then, the rest of the color samples are selected by 312 applying the former selection criterion to this starting point 313 for all sizes considered. The process is repeated five times to 314 provide five random training sets for each size, which consist 315 of very different subsets of the CCDC color samples. The 316 comparison of results obtained using these training sets en- 317 ables us to analyze the influence of the training set on the 318 system's performance depending on its size.

## RESULTS

320
Next, we present the results of the two analyses performed 321 for the colorimetric and the multispectral configurations of 322 the CCD camera-based imaging system.

## Colorimetric Configuration

Analysis I: Initial Fixed Color Sample 325
As expressed earlier, the first useful CCDC color sample (the 326 B2 color sample) is used in this analysis as the initial fixed 327 color sample. Starting from this point, the rest of the color 328 samples in the training sets, for all sizes considered, are se- 329 lected row by row from the color samples of the CCDC 330 chart by applying the selection criterion.


Figure 6. Colorimetric configuration: percentage of fluctuation of (a) the mean CIELAB color difference values, and (b) the mean RMSE values between results obtained using the five random training sets, depending on the number of samples in the training set. The CCCR chart was used as the test set.

The results obtained in this analysis indicate the ten333 dency of the system's performance depending on the num334 ber of samples in the training set, assuming that the behav335 ior of the system's performance is similar when we consider 336 training sets of different sizes selected from a different initial 337 color sample. The validity of this assumption and the influ338 ence of the training set on the system's performance are 339 studied in analysis II.
340 There is a limit to the improvement in the accuracy of 341 color measurement and spectral reconstruction when the 342 number of color samples in the training set is increased 343 (Figure 3, Table II). Improvement appears to be negligible 344 when there are more than about 60 color samples in the 345 training set. This is consistent with previous studies. ${ }^{11,14,21}$
346 These results prove that there are a relatively low mini347 mum and/or sufficient number of color samples for the 348 training set of the imaging system. After this point, increases 349 in the number of samples in the training set do not lead to 350 noticeable improvements in the system's performance.

## 351 Analysis II: Initial Randomly Selected Color Sample

352 In this second analysis, the initial color sample is randomly 353 selected from the CCDC chart. Starting from this point, the

Table III. Colorimetric configuration: mean, minimum, maximum, and standard deviation of the CIELAB color difference and the RMSE values obtained using the five random training sets from the CCDC chart, with 110 samples. The CCCR chart was used as the test set. The corresponding percentages of fluctuation are also given.

|  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Mean $\Delta E_{a b}^{*}$ | $\min \Delta E_{a b}^{*}$ | $\max \Delta E_{a b}^{*}$ | Std. dev $\Delta E_{a b}^{*}$ |
| Random 1 | 5.535 | 2.201 | 13.737 | 3.322 |
| Random 2 | 5.628 | 2.210 | 13.794 | 3.333 |
| Random 3 | 5.616 | 2.169 | 13.765 | 3.325 |
| Random 4 | 5.552 | 2.165 | 13.747 | 3.325 |
| Random 5 | 5.613 | 2.168 | 13.752 | 3.332 |
| Mean | 5.589 | 2.183 | 13.759 | 3.327 |
| Std. dev | 0.043 | 0.021 | 0.022 | 0.005 |
| \%fluctuation | 0.761 | 0.971 | 0.159 | 0.145 |
|  |  |  |  |  |
|  | Mean RMSE | Min RMSE | Max RMSE | Std.dev RMSE |
| Random 1 | $5.545 \times 10^{-2}$ | $3.240 \times 10^{-2}$ | $1.705 \times 10^{-1}$ | $2.885 \times 10^{-2}$ |
| Random 2 | $5.569 \times 10^{-2}$ | $3.273 \times 10^{-2}$ | $1.706 \times 10^{-1}$ | $2.872 \times 10^{-2}$ |
| Random 3 | $5.571 \times 10^{-2}$ | $3.284 \times 10^{-2}$ | $1.706 \times 10^{-1}$ | $2.870 \times 10^{-2}$ |
| Random 4 | $5.550 \times 10^{-2}$ | $3.234 \times 10^{-2}$ | $1.705 \times 10^{-1}$ | $2.882 \times 10^{-2}$ |
| Random 5 | $5.574 \times 10^{-2}$ | $3.313 \times 10^{-2}$ | $1.707 \times 10^{-1}$ | $2.871 \times 10^{-2}$ |
| Mean | $5.562 \times 10^{-2}$ | $3.269 \times 10^{-2}$ | $1.706 \times 10^{-1}$ | $2.876 \times 10^{-2}$ |
| Std. dev | $1.329 \times 10^{-4}$ | $3.257 \times 10^{-4}$ | $8.367 \times 10^{-5}$ | $6.964 \times 10^{-5}$ |
| \%fluctuation | 0.239 | 0.996 | 0.049 | 0.242 |

rest of the color samples in the training sets are selected row ${ }^{354}$ by row from the CCDC color samples by applying the selec- 355 tion criterion for all sizes considered. Five initial color 356 samples are selected randomly and used to generate the sub- 357 sequent random training sets for the different sizes consid- 358 ered.

359
Slightly different accuracies in the system's performance 360 are obtained using the five different random training sets for 361 fewer than about 80 color samples (Figure 4). The system's 362 performance fluctuates when the five different random 363 training sets of the same size are used. This is due to the 364 system's dependence on the training set, which tends to con- 365 verge for the equally sized five different random training sets 366 when the number of color samples is above 80 . 367

The dependence of the system's performance (accuracy 368 of color measurement and spectral reconstruction) on the 369 training set is clearly seen through a direct comparison of 370 the results obtained using the five different random training 371 sets for the different sizes considered (Figure 5). As can be 372 observed, the smaller the number of samples in the training 373 set, the greater the fluctuations between the CIELAB color 374 difference and the RMSE values obtained using the five ran- 375 dom training sets.

It is difficult to determine the exact number of samples 377 in the training set above which the system's performance is 378 independent of the set and of the number of color samples. 379 Consequently, taking into account several parameters calcu- 380 lated from the individual results for each of the five random 381
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Figure 7. Multispectral configuration: (a) mean CIELAB color difference and (b) mean RMSE values plotted versus the number of color samples in the training set (initial fixed color sample).
training sets-specifically the mean, minimum, maximum, ${ }^{382}$ and standard deviation of the CIELAB color differences and 383 the RMSE values. Percentages of fluctuation defined as fol- 384 lows have been computed for each size:

$$
\begin{equation*}
\text { \%Fluctuation }=100 \frac{\text { std. } \operatorname{dev}(\text { mean })}{\text { mean }} . \tag{7}
\end{equation*}
$$

386
For instance, if the mean CIELAB color difference is 387 analyzed, std. dev is the standard deviation of the mean 388 color difference (mean) of the five random training sets. If 389 the minimum color difference is considered, std. dev. is the 390 standard deviation of the mean minimum color difference 391 (mean) of the five random training sets. The same analysis 392 has been carried out for the maximum color difference and 393 the standard deviation, thus providing four percentages 394 which account for the fluctuation of the system depending 395 on the size of the training set used. The same analysis has 396 been carried out for RMSE values, providing four additional 397 percentages of fluctuation. Therefore, eight percentages of 398 fluctuation are obtained for each size of the training set. 399

These percentages are used to establish the minimum 400 and/or sufficient number of color samples necessary to 401 properly train any imaging system. Specifically, the criterion 402 used in this paper has been chosen as the minimum number 403 of color samples that provides all eight percentages of fluc- 404 tuation below $1 \%$. However, this criterion can be suitably 405 modified depending on the color and spectral accuracy 406

Table IV. Multispectral configuration: mean, minimum, maximum and standard deviation of the CIELAB color difference and the RMSE values obtained using different sized training sets (initial fixed color sample) from the CCDC chart. The CCCR chart was used as the test set.

|  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No. of color samples | Mean $\Delta E_{a b}^{*} \min \Delta E_{\text {ab }}^{*}$ | $\max \Delta E_{\text {ab }}^{*}$ | Std. dev $\Delta E_{\text {ab }}^{*}$ | Mean RMSE | Min RMSE | Max RMSE | Std. dev RMSE |
| 10 | 32.756 | 2.704 | 124.469 | 34.993 | $1.469 \times 10^{-1}$ | $1.541 \times 10^{-2}$ | $5.474 \times 10^{-1}$ |
| $1.379 \times 10^{-1}$ |  |  |  |  |  |  |  |
| 20 | 5.072 | 1.524 | 14.669 | 3.099 | $6.050 \times 10^{-2}$ | $2.166 \times 10^{-2}$ | $1.616 \times 10^{-1}$ |
| $3.235 \times 10^{-2}$ |  |  |  |  |  |  |  |
| 30 | 6.712 | 1.044 | 31.116 | 8.048 | $6.020 \times 10^{-2}$ | $2.396 \times 10^{-2}$ | $1.546 \times 10^{-1}$ |
| $3.642 \times 10^{-2}$ |  |  |  |  |  |  |  |
| 40 | 5.722 | 1.662 | 21.737 | 4.346 | $5.335 \times 10^{-2}$ | $2.683 \times 10^{-2}$ | $8.531 \times 10^{-2}$ |
| $1.708 \times 10^{-2}$ |  |  |  |  |  |  |  |
| 50 | 5.627 | 1.679 | 24.508 | 4.880 | $5.033 \times 10^{-2}$ | $2.572 \times 10^{-2}$ | $9.066 \times 10^{-2}$ |
| $1.654 \times 10^{-2}$ |  |  |  |  |  |  |  |
| 60 | 5.135 | 1.564 | 21.397 | 4.177 | $4.755 \times 10^{-2}$ | $2.269 \times 10^{-2}$ | $7.803 \times 10^{-2}$ |
| $1.485 \times 10^{-2}$ |  |  |  |  |  |  |  |
| 70 | 5.346 | 1.682 | 23.328 | 4.624 | $4.870 \times 10^{-2}$ | $2.063 \times 10^{-2}$ | $8.471 \times 10^{-2}$ |
| $1.642 \times 10^{-2}$ |  |  |  |  |  |  |  |
| 80 | 5.308 | 1.633 | 23.437 | 4.645 | $4.865 \times 10^{-2}$ | $1.996 \times 10^{-2}$ | $8.691 \times 10^{-2}$ |
| $1.676 \times 10^{-2}$ |  |  |  |  |  |  |  |
| 90 | 5.218 | 1.587 | 22.196 | 4.353 | $4.890 \times 10^{-2}$ | $2.099 \times 10^{-2}$ | $8.267 \times 10^{-2}$ |
| $1.569 \times 10^{-2}$ |  |  |  |  |  |  |  |
| 100 | 5.170 | 1.620 | 20.424 | 3.963 | $4.870 \times 10^{-2}$ | $2.271 \times 10^{-2}$ | $7.601 \times 10^{-2}$ |
| $1.475 \times 10^{-2}$ |  |  |  |  |  |  |  |
| 110 | 4.951 | 1.320 | 19.858 | 3.894 | $4.742 \times 10^{-2}$ | $2.400 \times 10^{-2}$ | $7.233 \times 10^{-2}$ |
| $1.389 \times 10^{-2}$ |  |  |  |  |  |  |  |
| 120 | 4.924 | 1.119 | 19.511 | 3.802 | $4.694 \times 10^{-2}$ | $2.371 \times 10^{-2}$ | $7.195 \times 10^{-2}$ |
| $1.400 \times 10^{-2}$ |  |  |  |  |  |  |  |
| 130 | 5.002 | 1.131 | 19.894 | 3.868 | $4.749 \times 10^{-2}$ | $2.378 \times 10^{-2}$ | $7.307 \times 10^{-2}$ |
| $1.382 \times 10^{-2}$ |  |  |  |  |  |  |  |
| 140 | 5.063 | 1.235 | 19.868 | 3.848 | $4.818 \times 10^{-2}$ | $2.464 \times 10^{-2}$ | $7.431 \times 10^{-2}$ |
| $1.389 \times 10^{-2}$ |  |  |  |  |  |  |  |
| 150 | 5.107 | 1.143 | 20.003 | 3.871 | $4.850 \times 10^{-2}$ | $2.429 \times 10^{-2}$ | $7.595 \times 10^{-2}$ |
| $1.411 \times 10^{-2}$ |  |  |  |  |  |  |  |
| 160 | 5.141 | 1.379 | 20.339 | 3.918 | $4.851 \times 10^{-2}$ | $2.380 \times 10^{-2}$ | $7.729 \times 10^{-2}$ |
| $1.443 \times 10^{-2}$ |  |  |  |  |  |  |  |
| 166 | 5.285 | 1.794 | 21.005 | 3.971 | $4.835 \times 10^{-2}$ | $2.380 \times 10^{-2}$ | $7.873 \times 10^{-2}$ |
| $1.457 \times 10^{-2}$ |  |  |  |  |  |  |  |

de Lasarte et al.: Influence of the number of samples of the training set on accuracy of color measurement and spectral reconstruction


Figure 8. Multispectral configuration: mean (a) CIELAB color difference and (b) RMSE values plotted versus the number of color samples in the training set for the five initial randomly selected color samples and the resulting training sets.
407 needed when the imaging system is used for any specific 408 industrial application.
409 Figure 6 shows the percentages corresponding to the 410 mean CIELAB color difference and the mean RMSE value as 411 a function of the size of the training set for the colorimetric 412 configuration. Taking into account the former criterion, the 413 number of color samples for which the percentage of fluc414 tuation is negligible (i.e., below 1\%) can be established as 415110 color samples for this configuration. The eight percent416 ages of fluctuation for this size of the training set are given 417 in Table III as well as the mean, minimum, maximum, and 418 standard deviation of the CIELAB color differences and the 419 RMSE values.
420 Taking all these results into account, it can be concluded 421 that the system's performance for the colorimetric configu422 ration seems to be independent of the training set used, in 423 terms of both the number of samples in the training set and 424 the training set itself, when the number of samples is above 425 110. These results hold in terms of accuracy of both color 426 measurement and spectral reconstruction.

## 427 Multispectral Configuration

428 Analysis I: Initial Fixed Color Sample
429 Similarly to the colorimetric configuration, in the multispec430 tral configuration there is a limit to the improvements in the


Figure 9. Multispectral configuration: (a) mean CIELAB color difference values and (b) mean RMSE values obtained using the five training sets selected from an initial randomly selected CCDC color sample, for all sizes of the training set considered. The CCCR color samples were used as the test set.
accuracy of color measurement and spectral reconstruction ${ }^{431}$ that can be brought about by increasing the number of color 432 samples in the training set. Improvements seem to be neg- 433 ligible when the training set is more than about 60 color 434 samples (Figure 7 and Table IV). This appears to be in agree- 435 ment with the findings of previous studies. ${ }^{11,14,21}$ The results 436 obtained in terms of CIELAB color differences and RMSE 437 values are slightly better in this case than those obtained 438 with the colorimetric configuration. This can be explained 439 by the larger experimental errors involved in the acquisition 440 sequence of the multispectral configuration, in which the 441 filters are mounted in a motorized filter wheel. The RGB 442 tunable filter used in the colorimetric configuration allows 443 faster and easier measurement performance.
Analysis II: Initial Randomly Selected Color Sample 445 With respect to different random training sets of the same 446 size, the system's performance for the multispectral configu- 447 ration fluctuates depending on the random training set used 448 when the sample sizes are low (Figure 8). However, the per- 449 formance seems to be similar for the five random training 450 sets that have 80 or more samples.


Figure 10. Multispectral configuration: percentage of fluctuation of the (a) mean CIELAB color difference values, and (b) mean RMSE values among the results obtained using the five random training sets, according to the number of samples in the training set. The CCCR chart was used as the test set.

The dependence of the system's performance (accuracy 453 of color measurement and spectral reconstruction) on the 454 training set is clearly seen through a direct comparison of 455 the results obtained using the five random training sets for 456 the different sizes considered (Figure 9).
457 Again, we observe that the smaller the number of 458 samples in the training set, the greater the fluctuations be459 tween the results obtained using the five random training 460 sets. With respect to the percentages of the CIELAB color 461 differences and RMSE values, the fluctuations in the mean, 462 minimum, maximum, and standard deviation are negligible 463 (below $1 \%$ ) when the number of samples in the training set 464 is greater than about 120 (Figure 10).
465 When these results are taken into account, an agreement 466 can be reached in terms of the accuracy of color measure467 ment and spectral reconstruction. With the criterion estab468 lished, we conclude that the system's performance is inde469 pendent of the training set used in terms of both the 470 number of samples in the training set and the training set 471 itself , when the number of samples is more than about 120 . 472 The eight percentages of fluctuation for this size of training 473 set are given in Table $V$ as well as the mean, minimum,

Table V. Multispectral configuration: mean, minimum, maximum and standard deviafion of the CIELAB color difference and the RMSE values obtained using the five random troining sets from the CCDC chart, with 120 color samples. The CCCR chart was used as the test set. The corresponding percentages of fluctuation are also given.

|  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Mean $\Delta E_{a b}^{*}$ | $\min \Delta E_{a b}^{*}$ | $\max \Delta E_{a b}^{*}$ | Std. dev $\Delta E_{\text {ab }}^{*}$ |
| Random 1 | 4.888 | 1.118 | 19.108 | 3.717 |
| Random 2 | 4.929 | 1.134 | 19.576 | 3.815 |
| Random 3 | 4.922 | 1.145 | 19.440 | 3.785 |
| Random 4 | 4.925 | 1.129 | 19.496 | 3.794 |
| Random 5 | 4.925 | 1.129 | 19.496 | 3.794 |
| Mean | 4.918 | 1.131 | 19.423 | 3.781 |
| Std. dev | 0.017 | 9.772 | 0.183 | 0.037 |
| \%fluctuation | 0.342 | 0.864 | 0.941 | 0.990 |
|  |  |  |  |  |
|  | Mean RMSE | Min RMSE | Max RMSE | Std. dev RMSE |
| Random 1 | $4.679 \times 10^{-2}$ | $2.383 \times 10^{-2}$ | $7.254 \times 10^{-2}$ | $1.401 \times 10^{-2}$ |
| Random 2 | $4.692 \times 10^{-2}$ | $2.363 \times 10^{-2}$ | $7.215 \times 10^{-2}$ | $1.407 \times 10^{-2}$ |
| Random 3 | $4.693 \times 10^{-2}$ | $2.375 \times 10^{-2}$ | $7.213 \times 10^{-2}$ | $1.400 \times 10^{-2}$ |
| Random 4 | $4.695 \times 10^{-2}$ | $2.378 \times 10^{-2}$ | $7.224 \times 10^{-2}$ | $1.402 \times 10^{-2}$ |
| Random 5 | $4.695 \times 10^{-2}$ | $2.378 \times 10^{-2}$ | $7.224 \times 10^{-2}$ | $1.402 \times 10^{-2}$ |
| Mean | $4.691 \times 10^{-2}$ | $2.375 \times 10^{-2}$ | $7.226 \times 10^{-2}$ | $1.402 \times 10^{-2}$ |
| Std. dev. | $6.723 \times 10^{-5}$ | $7.503 \times 10^{-5}$ | $1.645 \times 10^{-4}$ | $2.702 \times 10^{-5}$ |
| \%fluctuation | 0.143 | 0.316 | 0.228 | 0.193 |

maximum and standard deviation of the CIELAB color dif- ${ }^{474}$ ferences and the RMSE values.

CONCLUSIONS
In this article, we have analyzed the influence of the number 477 of samples in the training set on the accuracy of the color 478 measurement and spectral reconstruction. We have consid- 479 ered the imaging system's performance depending on the 480 size of the training set and on the specific training set for 481 each size. A method based on the calculation of percentages 482 of fluctuation according to the training set has been devel- 483 oped to establish the minimum and/or sufficient number of 484 color samples in the training set above which the system's 485 performance is independent of the number of samples. This 486 increases the robustness of the colorimetric and spectral 487 characterizations that are often used in the industry, which 488 are based on establishing a direct relationship between the 489 digital levels of the response of the imaging system and the 490 corresponding tristimulus values or, equivalently, the reflec- 491 tance spectra.

Two configurations of a CCD camera-based imaging 493 system have been used for this purpose: a colorimetric con- 494 figuration with three acquisition channels and a multispec- 495 tral configuration with seven acquisition channels. The re- 496 sults suggest that the system's performance seems to be 497 independent of the training set, in terms of both the number 498 of samples in the training set and the training set itself, when 499 there are over 110 samples for the colorimetric configuration 500 and over 120 samples for the multispectral configuration. 501

502 This result is true when percentages of fluctuation below $1 \%$ 503 are considered. The results hold in terms of accuracy of 504 color measurement and spectral reconstruction and can be 505 considered as the minimum and/or sufficient number of 506 color samples in the training set for the two configurations 507 of the imaging system. However, depending on the level of 508 accuracy of color and spectral assessments that is required of 509 the imaging system, the chosen criterion can be suitably 510 modified to obtain the size of the training set that is needed 511 for a specific industrial application. Furthermore, the meth512 odology can also be used for suitable color samples other 513 than those included in this article, depending on the indus514 trial process under consideration.

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