B-spline basis for adaptive piezoelectric mirror shape reconstruction

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1. Introduction

For adaptive systems having many sensors and actuators local control methods are believed to provide higher speed of correction than the classical Zernike approach [1]. A local wavefront reconstruction technique is needed to provide a feedback loop from sensor data of a wavefront zone to a corresponding actuator. The cubic B-spline basis has shown good reconstruction quality for complex-shaped wavefronts [2].

A B-spline wavefront representation has the additional advantage of being formally similar to the classical Zernike basis.

Wavefront in B-spline basis is built as a product of 1-D B-splines.

$$W(x,y)=\sum_{i=0}^{n}\sum_{j=0}^{m}a_{ij}B_{i,k}(x)B_{j,l}(y) \qquad (1)$$

$$B_{i,k}(x)=\frac{x-t_i}{t_{i+k-1}-t_i}B_{i,k-1}(x)+\frac{t_{i+k}-x}{t_{i+k}-t_{i+1}}B_{i+1,k-1}(x)$$
$$B_{i,1}(x)=\begin{cases}1, & t_i\le x\le t_{i+1}\\ 0, & otherwise\end{cases} \qquad (2)$$

meaning each B-spline $B_i(x)$ is a polynomial which extends over a limited number of 1D regions across the wavefront, and zero elsewhere. Each 2-D B-spline $B_i(x)B_j(y)$ is defined within its corresponding subdomain and centered onto a so-called control point, with height defined by coefficient a_{ij}.

Unknown coefficients a_{ij} for best approximation can be determined using least-squares method.

$$a = B^{-1}\cdot W \qquad (3)$$

Experimental wavefront shapes obtained with a 37-actuator piezoelectric deformable mirror (OKO Technologies) and measured with a 127 data points Shack-Hartmann sensor were reconstructed using B-splines of different orders and with different numbers of control points. RMS fitting error for B-spline reconstruction is then compared to a traditional Zernike reconstruction of equivalent numbers of modes.

Coefficients for Zernike and B-spline approximations were determined using singular value decomposition (SVD) procedure. Number of modes for SVD is chosen to avoid ill-conditioning for Zernike fitting.

Control points (and subsequently, the subdomain array) are positioned in a square array covering the complete pupil. Edge control points are chosen to be beyond the pupil area bounds (Fig.1)

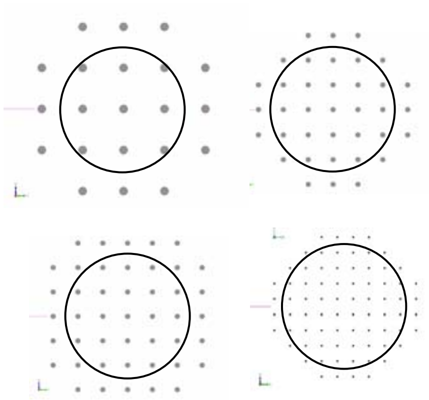


Figure 1: Position of control points for 21, 37, 45 and 76 B-spline basis

2. Results

For studied wavefronts (Tab.1) cubic B-spline reconstruction shows equal or better RMS results compared to Zernike of equivalent number of modes.

B-spline reconstruction has thus advantages for complex-shaped wavefronts.

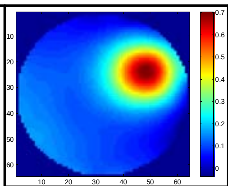
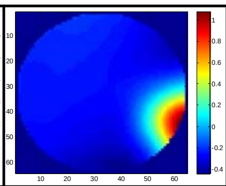
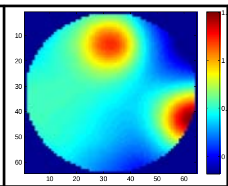
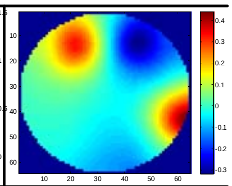
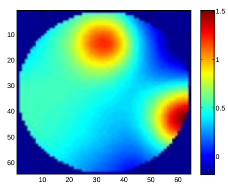
Number of basis functions for Zernike (B-spline) decomposition B-spline order =4								
	RMS error		RMS error		RMS error		RMS error	
	Zernike	B-spline	Zernike	B-spline	Zernike	B-spline	Zernike	B-spline
20 (21)	0.2124	0.1756	0.1766	0.1139	0.3638	0.2650	0.1371	0.1329
35 (37)	0.1140	0.0931	0.0902	0.0693	0.1890	0.1140	0.0820	0.0518
44 (45)	0.0873	0.0914	0.0682	0.0452	0.1426	0.0936	0.0619	0.0464
77 (76)	0.0557	0.0577	0.0341	0.0364	0.0964	0.0852	0.0443	0.0306

Table 1: RMS fitting error for Zernike and cubic B-splines reconstruction

RMS error for B-spline fitting depends on used B-spline basis function order (Tab.2). With B-spline order equal to the maximum number of control points in one direction, all B-spline basis functions cover whole pupil area and thus become global and RMS error has its minimum. Order reduction leads to basis functions localization and fitting quality decreasing.



	Zernike	B-sp 4	B-sp 5	B-sp 6	B-sp 7
20 (21)	0.3638	0.2650	0.2641	0.2650	0.2663
35 (37)	0.1890	0.1140	0.1059	0.1034	0.1040
44 (45)	0.1426	0.0936	0.0841	0.0794	0.0788
77 (76)	0.0964	0.0852	0.0739	0.0685	0.0644

Table 2: RMS fitting error for Zernike and B-splines of 4-7th order reconstruction

3. Conclusion

Using zonal B-spline basis reconstruction for a complex wavefront with multiple local peaks does not lead to residual error increasing while makes it possible to implement local or combined control algorithms and thus can speed up correction in both active and adaptive systems.

4. References

[1] D.G.MacMartin, “Local, hierarchic, and iterative reconstructors for adaptive optics”, J. Opt. Soc. Am. A 20, 1084-1093 (2003)

[2] M.Ares, S.Royo, “Comparison of cubic B-spline and Zernike fitting techniques in complex wavefront reconstruction”, Appl.Opt., v 45 (27), 6954-6964, 2006